Multi-way Communication and Cooperation

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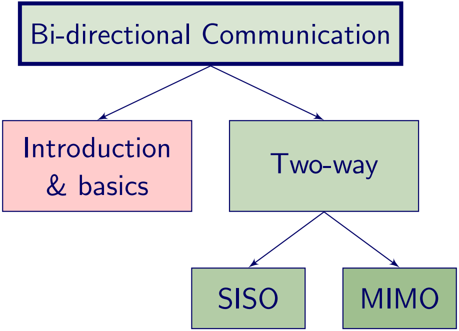
RUB, Bochum, Germany

Structure


Bi-directional Communication
Introduction & basics
Two-way
SISO
MIMO

Part 1: Intro. & Basics

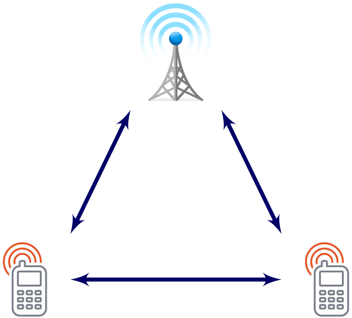
Outline

1. What is bi-directional communication?
2. Why multi-way?
3. History
   - Point-to-point
   - Multiple-access channel
   - Broadcast channel
   - Relay channel

What is bi-directional Communications

Definition

Nodes acting as sources and destinations simultaneously.



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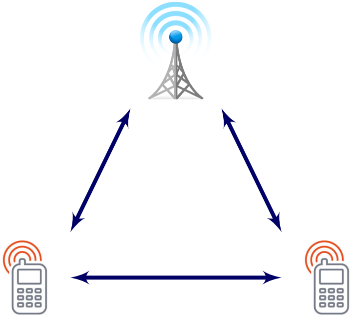

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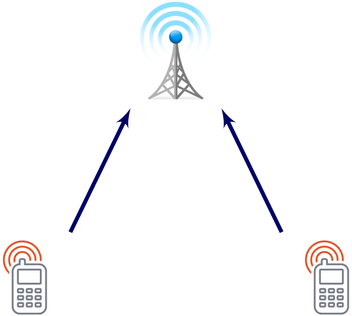

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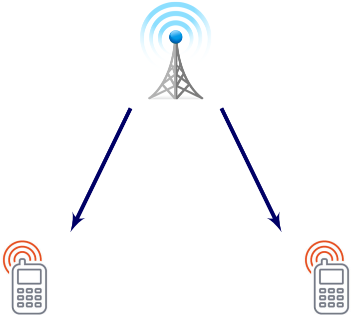

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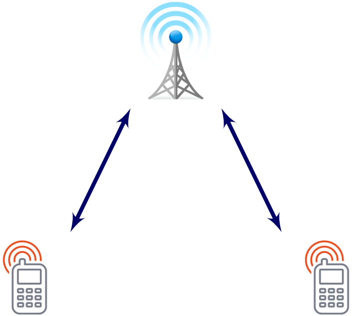

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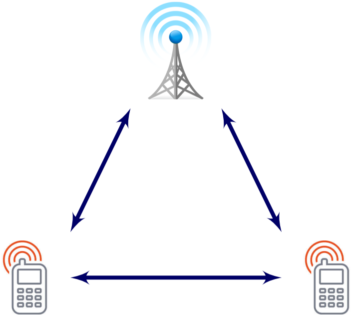

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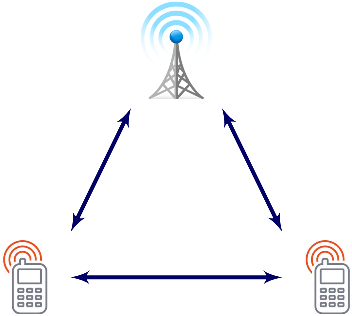

Goal

Introduce and discuss techniques for bi-directional communications.

Outline

1. What is bi-directional communication?
2. Why multi-way?
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Changing Game

Rapid changes in communications: applications, services, requirements, etc.

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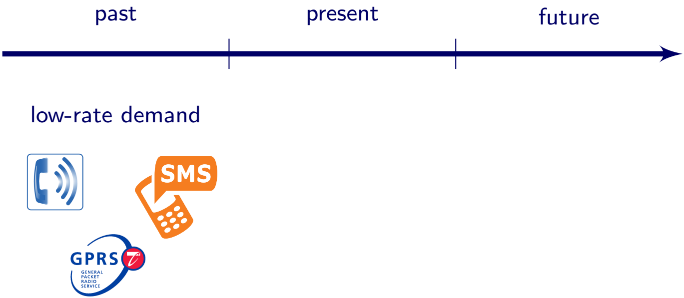

Sources: experian.com, allmytech.pk, en.wikipedia.org, ecnmag.com, kpcb.com, play.google.com

Changing Game

Rapid changes in communications: applications, services, requirements, etc.

past | present | future

low-rate demand | high-rate demand

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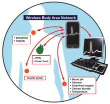

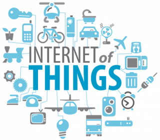

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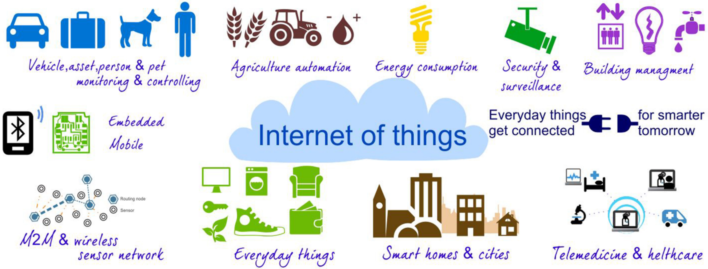
Changing Game

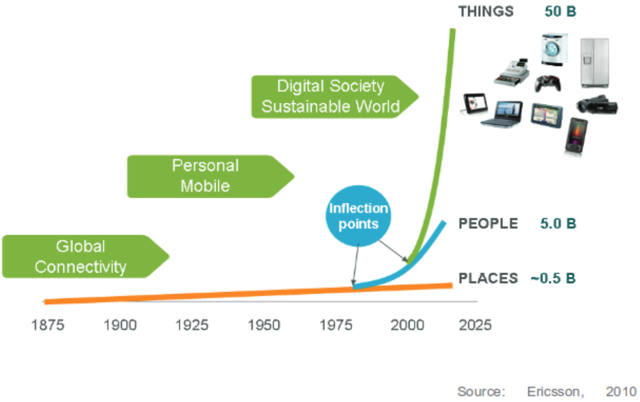
Future: Everything can communicate!



New players, new rules!

Sources: influxis.com

Towards 50B devices in 2020!



Increasing number of connected devices (IoT, M2M, etc.)

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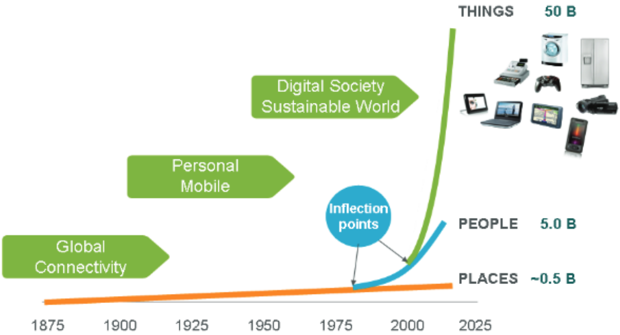
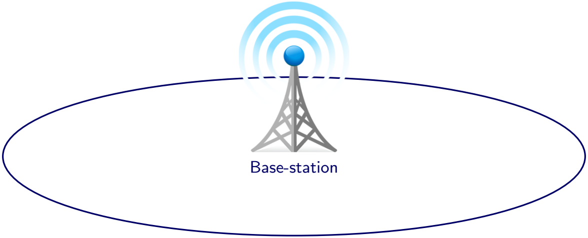


Source: Ericsson, 2010

Increasing number of connected devices (IoT, M2M, etc.)

Consequence: Networks must support much higher data-rates

Ideas



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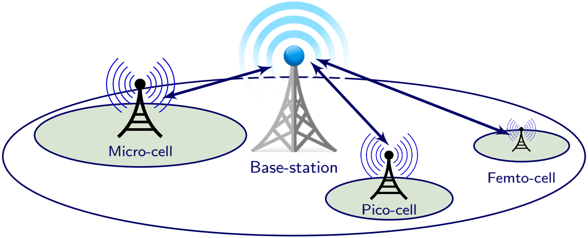

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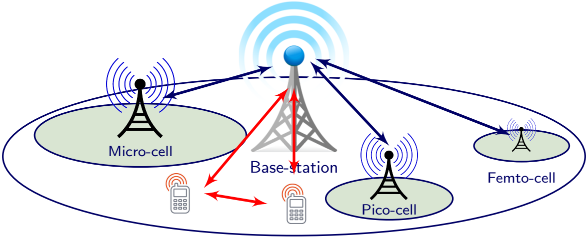

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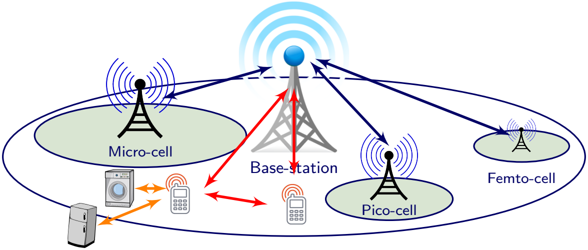

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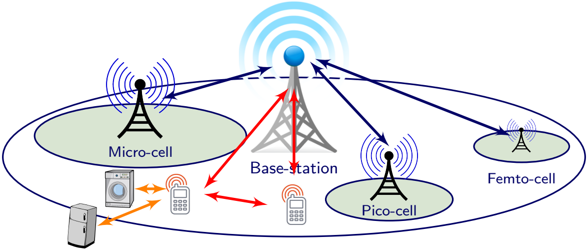

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- Important: Most communication is bi-directional (uplink/downlink, feedback, etc.)

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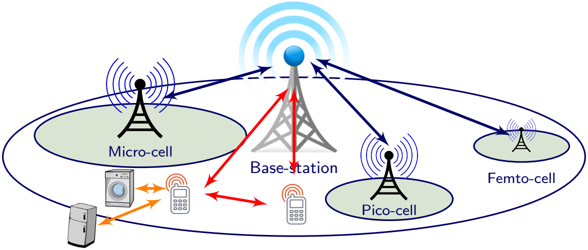

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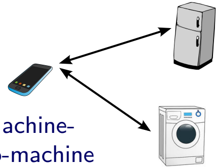
$\Rightarrow$ need to study bi-directional communication

More sophisticated network topologies!



Machine-
to-machine
Point↔multi-point

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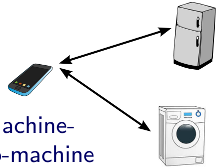

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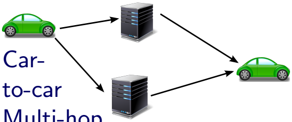

Car-
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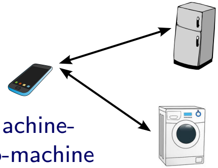

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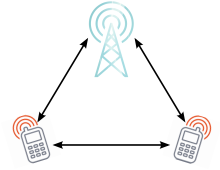

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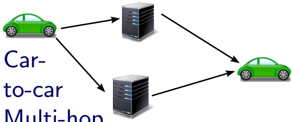

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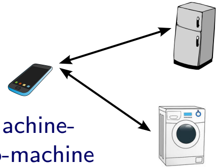

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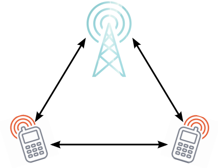

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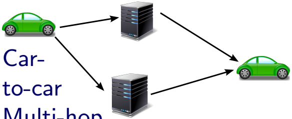

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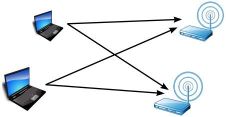

X-Channel

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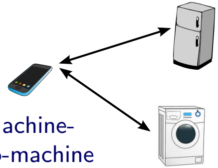

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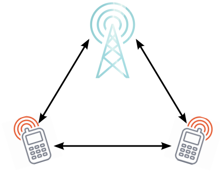

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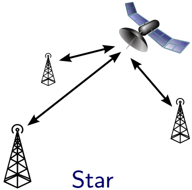

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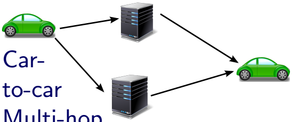

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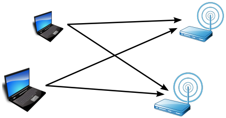

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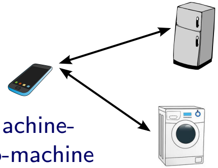

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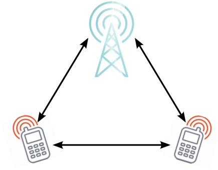

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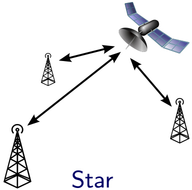

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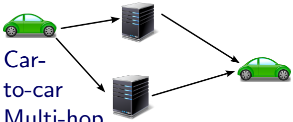

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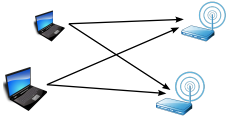

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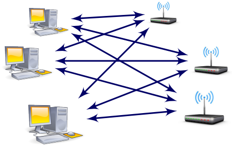

Interference
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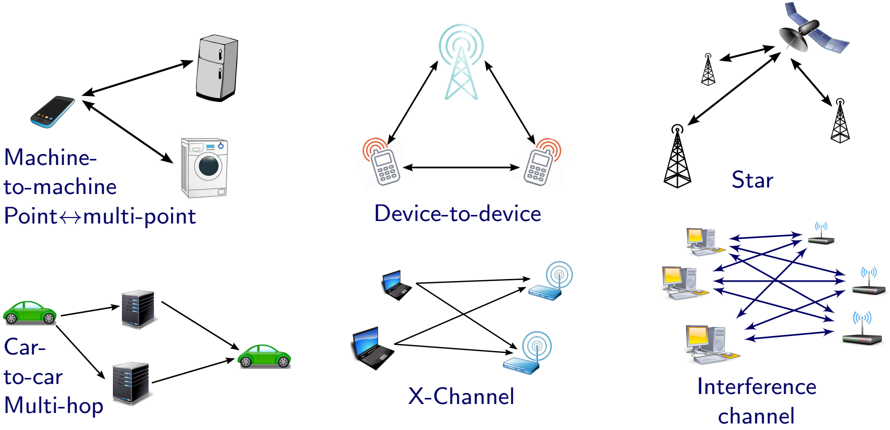
More sophisticated network topologies!



- Important factors: Multi-way communications and Relaying!

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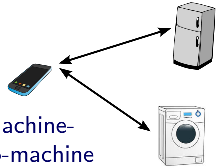

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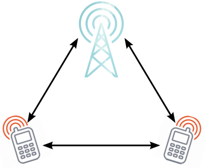

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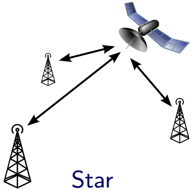

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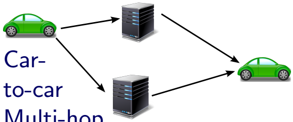
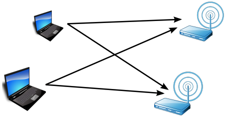


Car-to-car
Multi-hop



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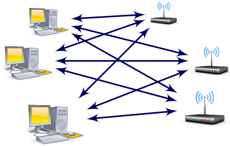
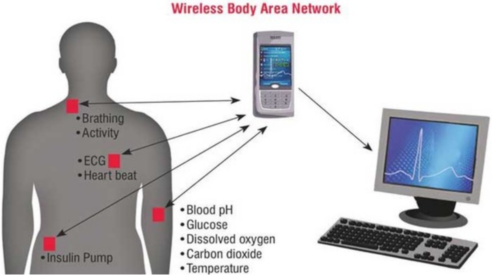


Interference channel

- Important factors: Multi-way communications and Relaying!

⇒ need to study bi-directional communication

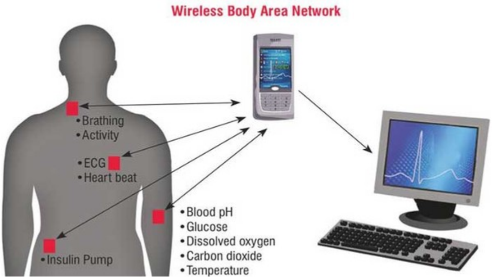
Body-area networks



- Multiple sensors communicating with a central node,

Sources: www.examiner.com

Body-area networks



- Multiple sensors communicating with a central node,
- Higher spectral/power efficiency $\Rightarrow$ shorter transmission duration $\Rightarrow$ longer life-cycle/less radiation

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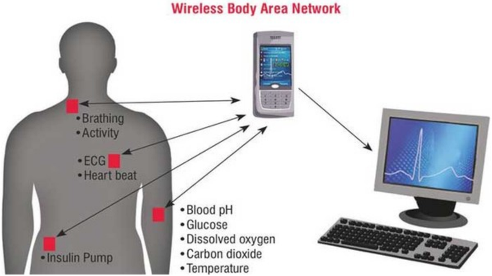

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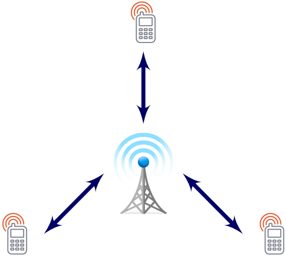

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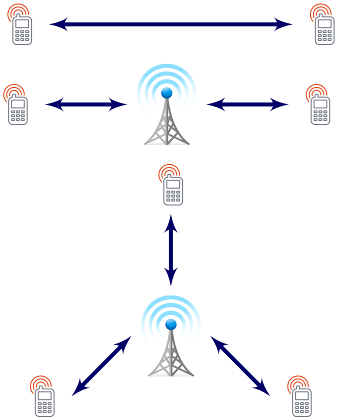

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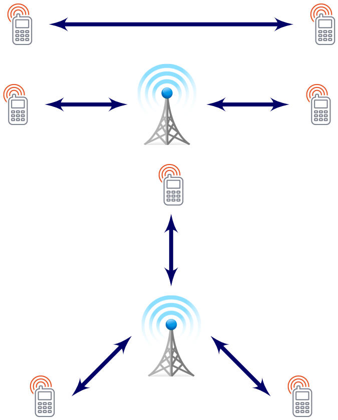

One-way communication is an integral part of two-way communication

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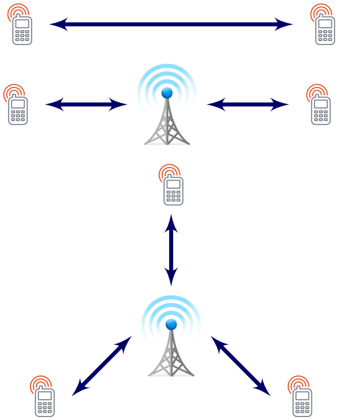

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Brief Review

Outline

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2. Why multi-way?

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   - Multiple-access channel
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   - Relay channel

History

In the beginning... One-way (uni-directional) Communications:

- Point-to-point (P2P): [Shannon 48],

History

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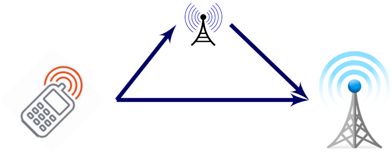
- Point-to-point (P2P): [Shannon 48],
- Relay channel (RC): [van der Meulen 71], [Cover & El-Gamal 79],



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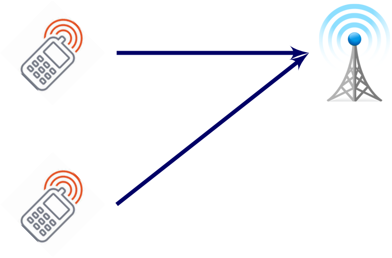

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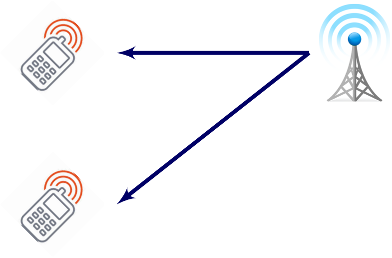

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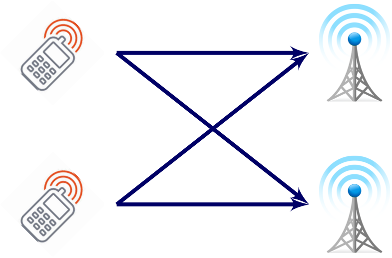

P2P

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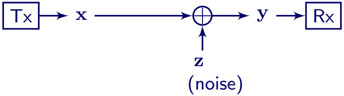

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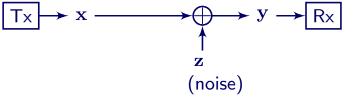

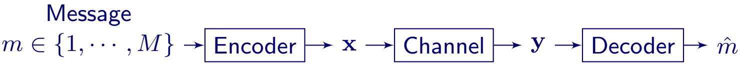

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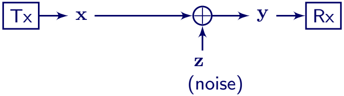

Message

$m \in \{1, \cdots, M\}$ → Encoder → $\mathbf{x}$ → Channel → $\mathbf{y}$ → Decoder → $\hat{m}$

- Error probability: $P_e = \text{Prob}(\hat{m} \neq m)$,

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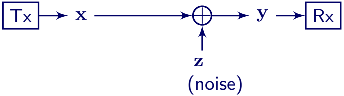

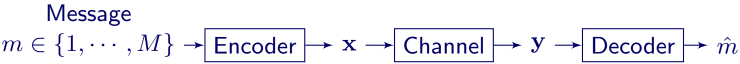

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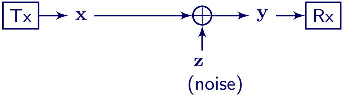

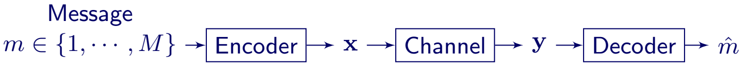

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P2P: Simple example

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Assume:

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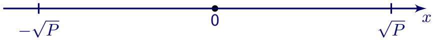

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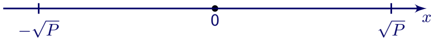

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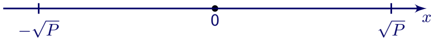

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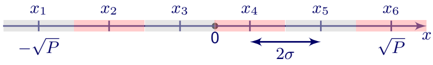

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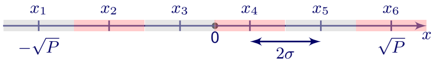

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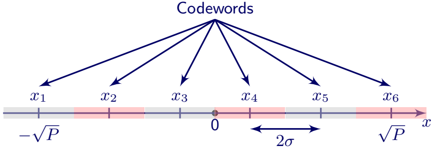

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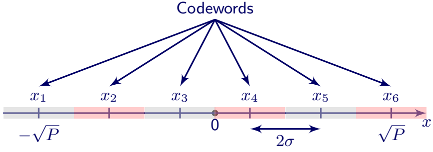

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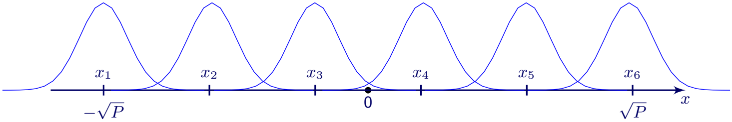
In reality, Gaussian noise power $\mathbb{E}[z^2] = \sigma^2$ (not $|z| < \sigma$).



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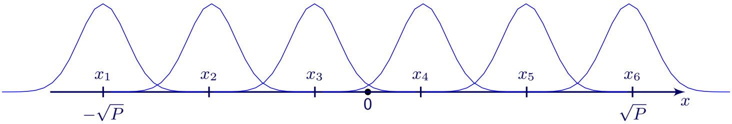

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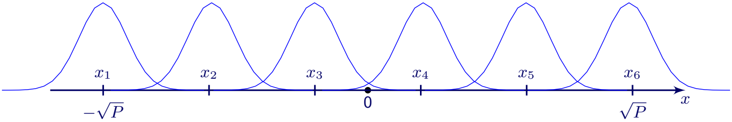

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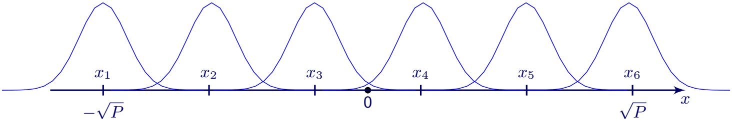

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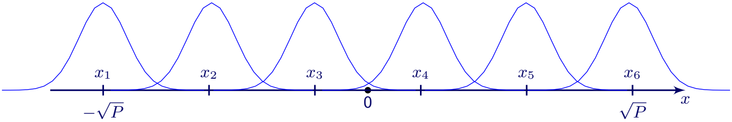

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- Capacity [Shannon 48].

MAC

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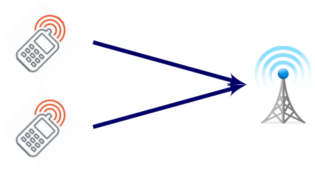

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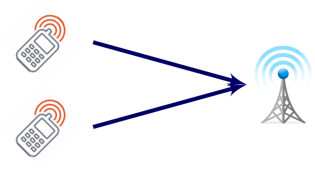

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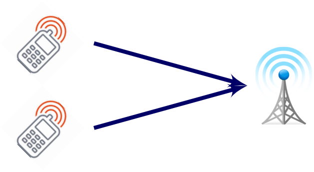

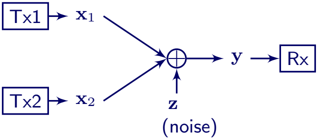

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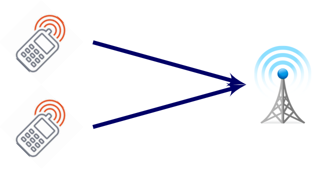

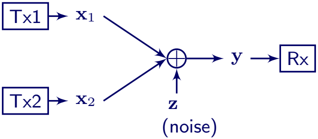

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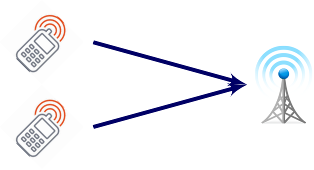

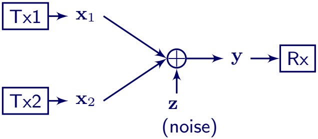

Rate-region

The rate-region is the set of achievable rate pairs (R_1, R_2).

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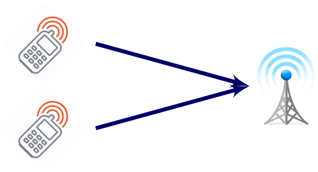
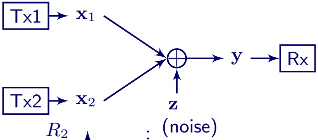
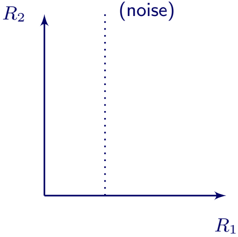
- Successive decoding:
- Treat $\mathbf{x}_2$ as noise $\Rightarrow$

$$R_1 = \frac{1}{2}\log\left(1 + \frac{P_1}{\sigma^2 + P_2}\right),$$







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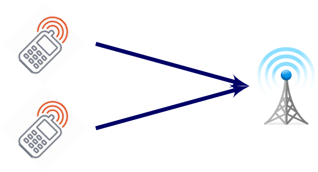

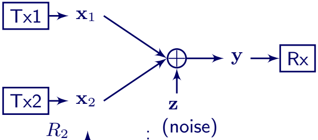

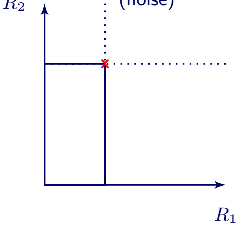

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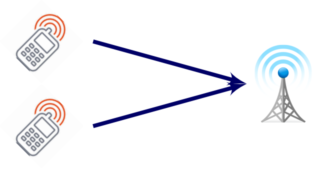

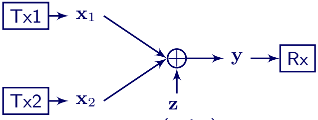

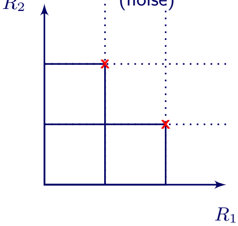

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- Subtract $\mathbf{x}_1$ and decode $\mathbf{x}_2$ $\Rightarrow$

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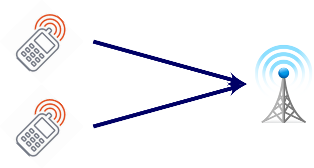
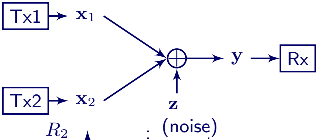
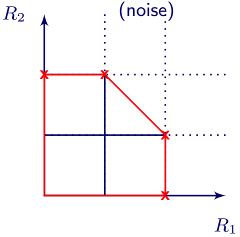
- opposite order,
- Capacity region [Ahlswede 71],

$$R_i \leq \frac{1}{2}\log\left(1 + \frac{P_i}{\sigma^2}\right), \quad i = 1, 2,$$

$$R_1 + R_2 \leq \frac{1}{2}\log\left(1 + \frac{P_1 + P_2}{\sigma^2}\right).$$







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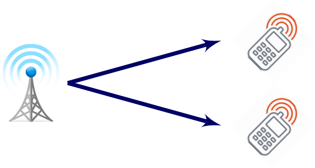

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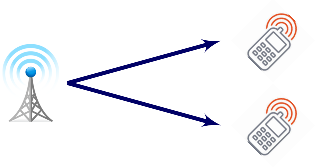

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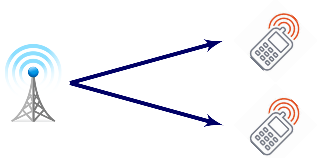

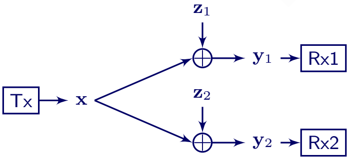

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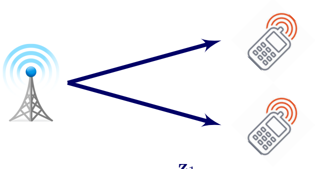
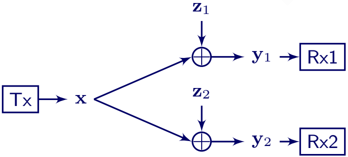
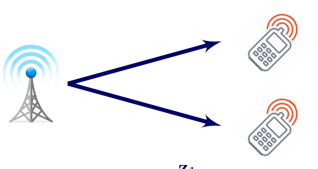
- Goal: Find the rate-region,

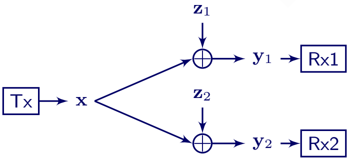




BC

- Superposition coding:
- Send $\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2$, powers $P_1 + P_2 = P$,





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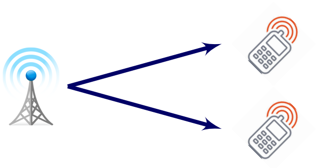
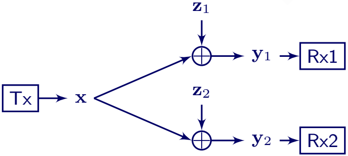
- Send $\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2$, powers $P_1 + P_2 = P$,
- Both receivers decode $\mathbf{x}_2$ treating $\mathbf{x}_1$ as noise $\Rightarrow$

$$R_2 = \frac{1}{2} \log \left(1 + \frac{P_2}{\sigma_2^2 + P_1} \right),$$





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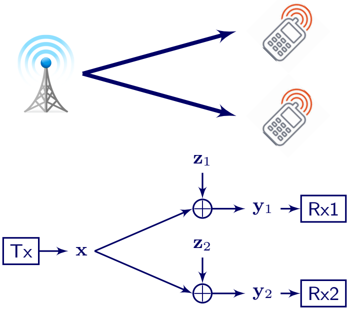

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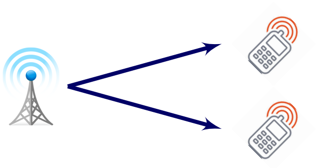
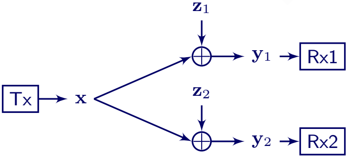
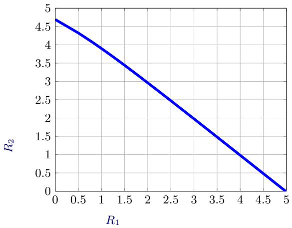
- Rate-region = Capacity region [Cover 72], [Bergmans 74],

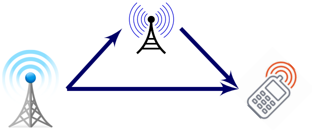






Relay Channel

one transmitter, one receiver, one relay:



Relay Channel

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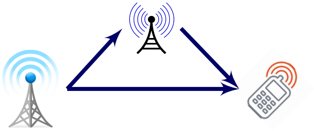
- Powers: P, P_r,



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one transmitter, one receiver, one relay:

- Inputs: $\mathbf{x}$, $\mathbf{x}_r$,
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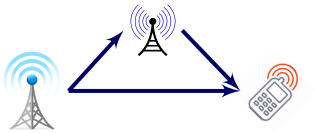

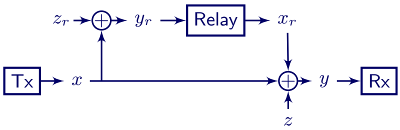

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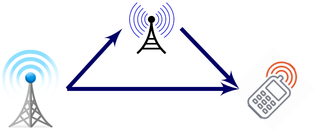
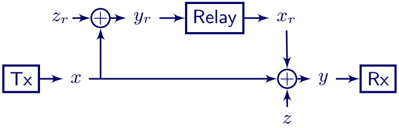
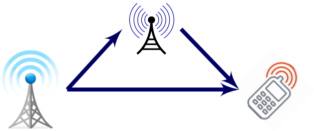
- Noise variance: σ^2 and σ_r^2.

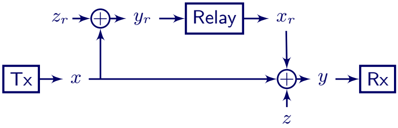




Relay Channel

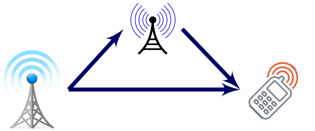
- Main schemes:
  Decode-forward,
  Compress-forward

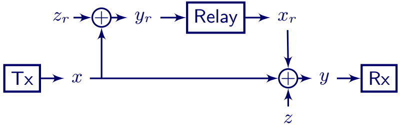




Relay Channel

- Main schemes: Decode-forward, Compress-forward
- Notice: Interaction between consecutive symbols/codewords at Rx!





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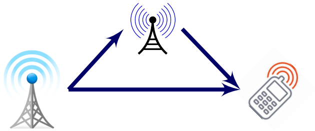
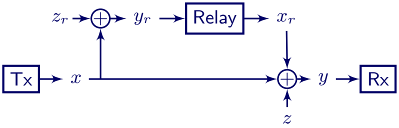
- Interaction can be exploited using block-Markov encoding [Cover & El-Gamal 79],





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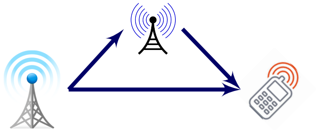
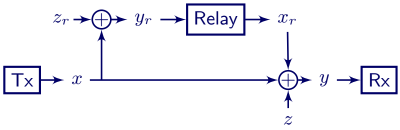
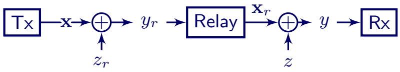
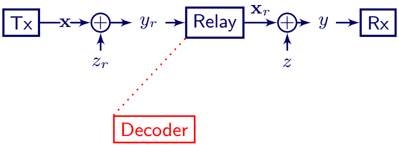
- Interaction can be exploited using block-Markov encoding [Cover & El-Gamal 79],
- For simplicity: Consider a separated relay channel







Relay Channel: DF

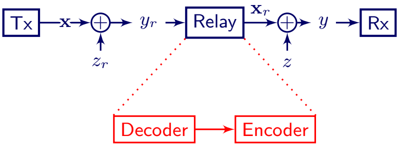


Decode-forward:

- Relay decodes $\mathbf{x}$ $\Rightarrow$

$$R \leq \frac{1}{2}\log\left(1+\frac{P}{\sigma_r^2}\right).$$

Relay Channel: DF



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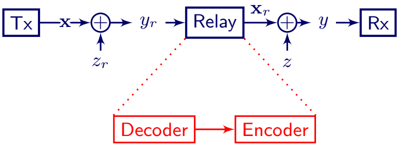

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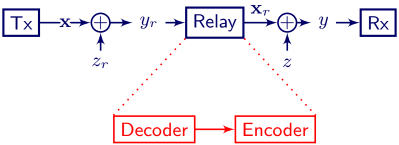

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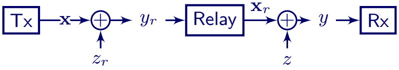
- Achievable rate $\min\left\{\frac{1}{2}\log\left(1+\frac{P}{\sigma_r^2}\right), \frac{1}{2}\log\left(1+\frac{P_r}{\sigma^2}\right)\right\}$.

Relay Channel: Cut-set bound



Cut-set Bound: Capacity bounded by the rate of information flow from a sub-set of nodes to the remaining nodes

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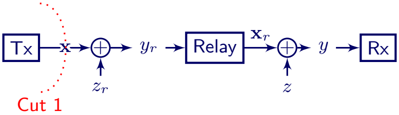

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- Cut 1 $\Rightarrow C \leq \frac{1}{2}\log\left(1+\frac{P}{\sigma_r^2}\right)$.

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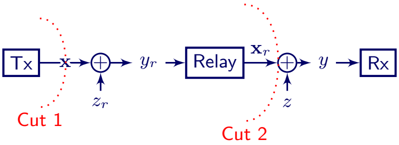

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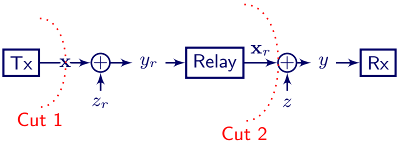

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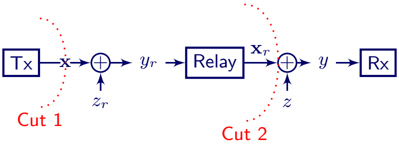

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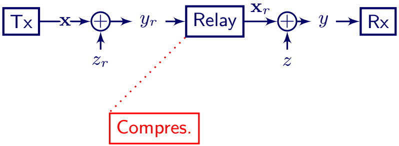
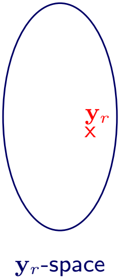
- DF Optimal: Coincides with the cut-set bound.

Relay Channel: CF



Compress-forward:

- Relay compresses/quantizes $\mathbf{y}_r$ to $\hat{\mathbf{y}}_r = \mathbf{y}_r + \mathbf{z}_c$ ($\mathbf{z}_c$ compression noise)



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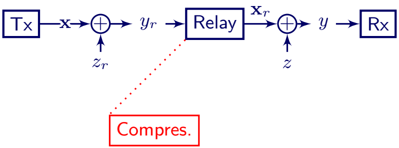

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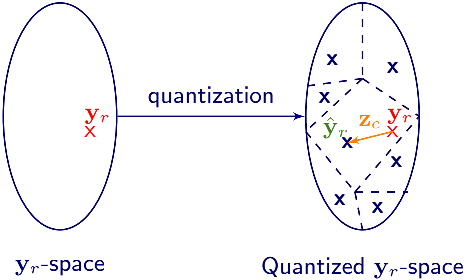

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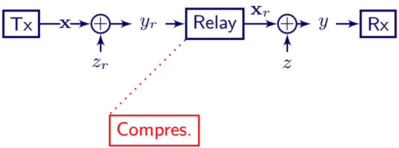

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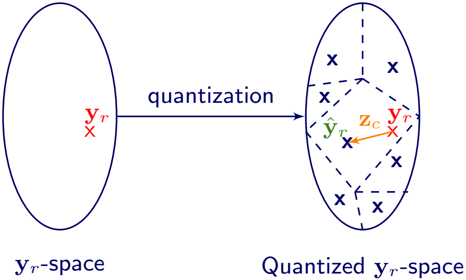
- Compression rate R_c ($\frac{1}{n}\log$(number of bins))



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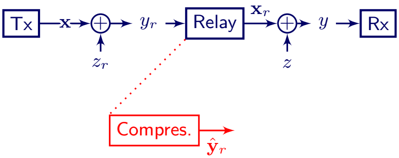
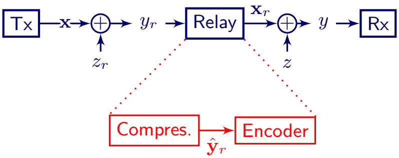


Compress-forward:

- Compression noise variance $D = (P + \sigma_r^2) \cdot 2^{-2R_c}$ (optimal rate-distortion)

Relay Channel: CF



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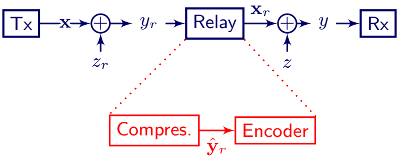

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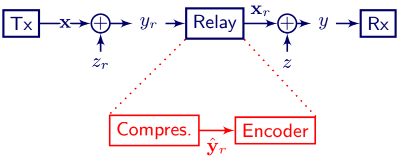

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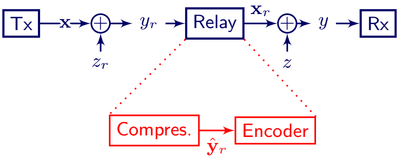

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- Rx obtains $\hat{\mathbf{y}}_r = \mathbf{x} + \mathbf{z}_r + \mathbf{z}_c$.
- Rx then decodes $\mathbf{x}$ from $\hat{\mathbf{y}}_r \Rightarrow$
  $R = \frac{1}{2}\log\left(1 + \frac{P}{\sigma_r^2 + D}\right) \leq \frac{1}{2}\log\left(1 + \frac{P(P_r+\sigma^2)}{\sigma_r^2(P_r+\sigma^2)+\sigma^2(P+\sigma_r^2)}\right)$.

Part 2: SISO Bi-directional

Outline

Two-way Channel

Channel with two transceivers: First studied by Shannon (1961)

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Two-way Channel

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- $P(X_1, X_2)$ vs. $P(X_1)P(X_2)$:
  - $P(X_1, X_2)$ allows interactive coding: X_i and Y_i can be dependent, $X_1 = \mathcal{E}(m_1, Y_1)$
  - $P(X_1)P(X_2)$ does not allow interactive coding: X_i and Y_i independent, $X_1 = \mathcal{E}(m_1)$

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Channel with two transceivers: First studied by Shannon (1961)

- Outer bound: Cut-set $R_i \leq I(X_i, Y_j|X_j)$ maximized over $P(X_1, X_2)$,
- Inner bound: P2P codebooks $R_i \leq I(X_i, Y_j|X_j)$ maximized over $P(X_1, X_2) = P(X_1)P(X_2)$,
- $P(X_1, X_2)$ vs. $P(X_1)P(X_2)$:
  - $P(X_1, X_2)$ allows interactive coding: X_i and Y_i can be dependent, $X_1 = \mathcal{E}(m_1, Y_1)$
  - $P(X_1)P(X_2)$ does not allow interactive coding: X_i and Y_i independent, $X_1 = \mathcal{E}(m_1)$
- Consequence: Bounds do not coincide

Two-way Channel

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$\Rightarrow$ unknown capacity!

Two-way Channel

- However: Bounds coincide if channel is separable!

Two-way Channel

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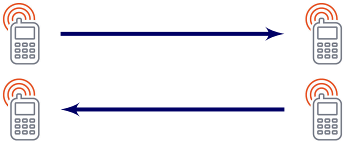

Two-way Channel

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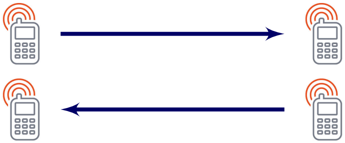

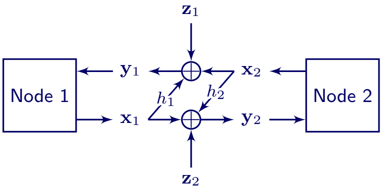

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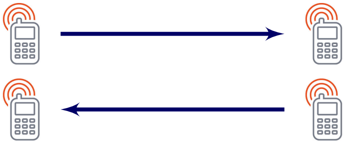

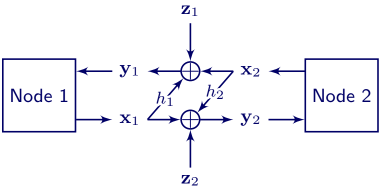

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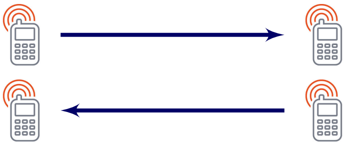

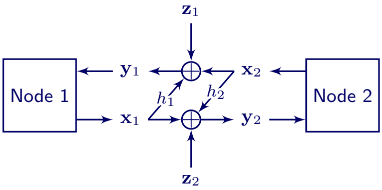

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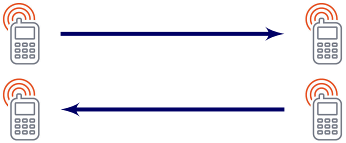
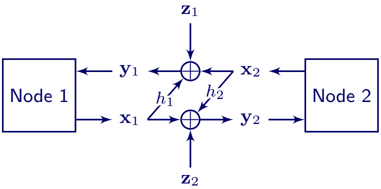
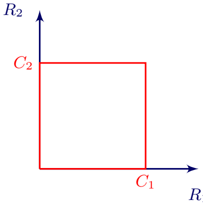
- Achievable rate: $R_i \leq \frac{1}{2}\log\left(1 + \frac{P_i}{\sigma_j^2}\right)$ (independent of self-interference!),







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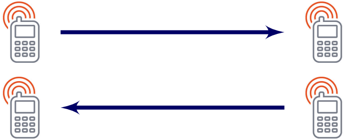
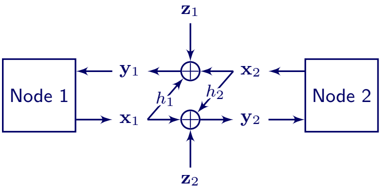
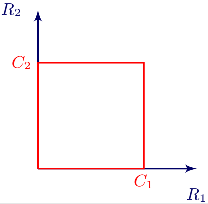
- Capacity [Han 84],





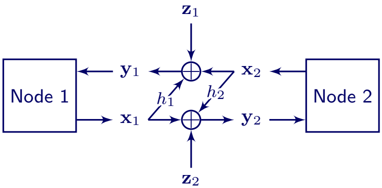


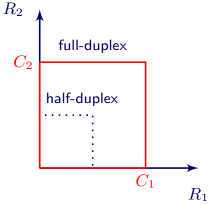
Two-way Channel

Remarks:

Half-duplex vs. full-duplex:

- Half-duplex:
  $R_i \leq \frac{1}{4} \log \left(1 + \frac{P_i}{\sigma_j^2}\right).$
- Full-duplex:
  $R_i \leq \frac{1}{2} \log \left(1 + \frac{P_i}{\sigma_j^2}\right).$
- Full-duplex achieves double rate.





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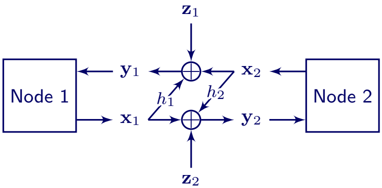
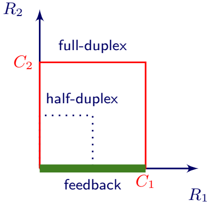


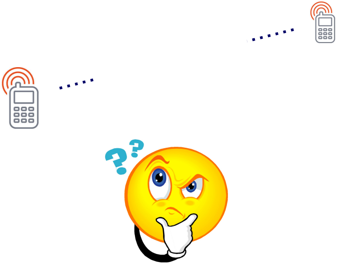
Feedback vs. Two-way:

- Feedback does not increase the P2P capacity [Shannon 56].
- Rate: $R_1 \leq \frac{1}{2}\log\left(1+\frac{P_1}{\sigma_2^2}\right)$.
- Two-way achieves double rate.



Two-way Channel

- What happens if nodes are far/physically separated?



Outline

1. Two-way channel

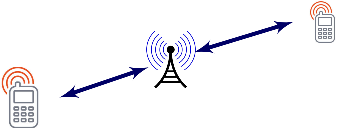
2. Two-way relay channel
   - The linear-deterministic approximation
   - Lattice codes

3. Multi-way relay channel
   - Multi-pair Two-way Relay Channel
   - Multi-way Relay Channel

4. Multi-way Channel

Two-way Relay Channel

Channel with two transceivers and a relay: First studied by [Rankov & Wittneben 06]

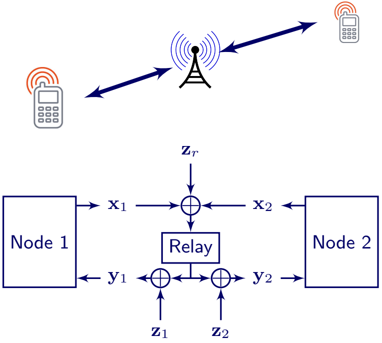


Two-way Relay Channel

Channel with two transceivers and a relay: First studied by [Rankov & Wittneben 06]

Gaussian two-way relay channel:

- Inputs: $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_r$ with powers P_1, P_2, and P_r,



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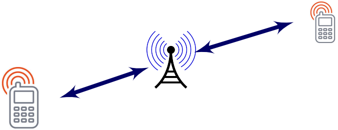

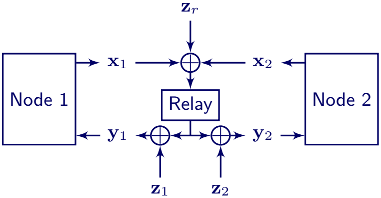

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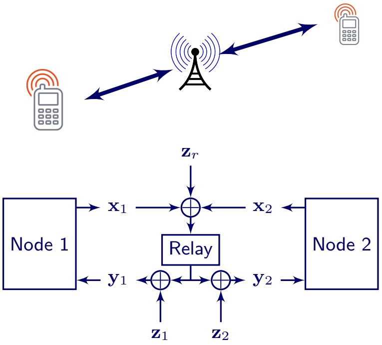
- Goal: Find the capacity region.

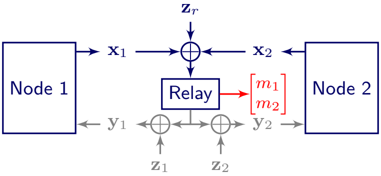


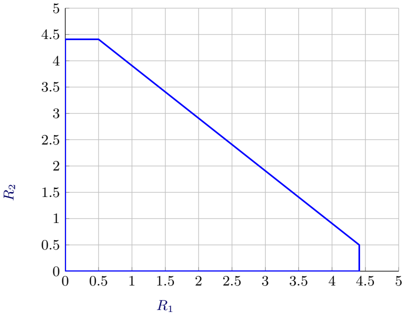
Classical approach

- Treat uplink as a MAC ⇒

$$R_i \leq \frac{1}{2}\log\left(1+\frac{P_i}{\sigma^2}\right)$$

$$R_1 + R_2 \leq \frac{1}{2}\log\left(1+\frac{P_1+P_2}{\sigma^2}\right),$$





Classical approach

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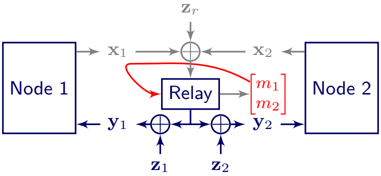
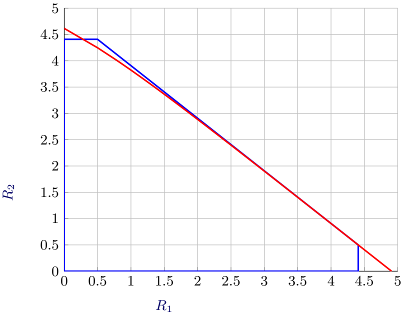
$$R_1 \leq \frac{1}{2}\log\left(1+\frac{p_1}{\sigma_1^2}\right)$$

$$R_2 \leq \frac{1}{2}\log\left(1+\frac{p_2}{\sigma_2^2+p_1}\right)$$

with $p_1+p_2 \leq P_r$,





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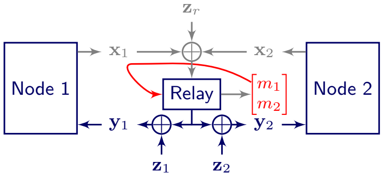
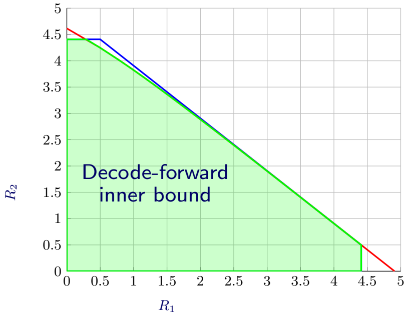
with $p_1+p_2 \leq P_r$,

- Achievable region: intersection





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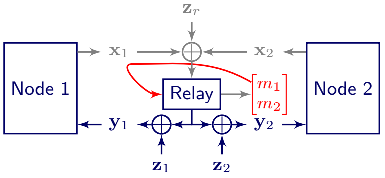
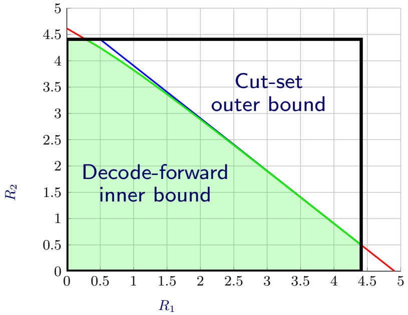
with $p_1+p_2 \leq P_r$,

- Achievable region: intersection
- Cut-set bound:
$R_i \leq$
$\min\left\{\frac{1}{2}\log\left(1+\frac{P_i}{\sigma^2}\right), \frac{1}{2}\log\left(1+\frac{P_r}{\sigma_2^2}\right)\right\}$





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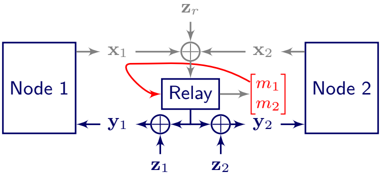
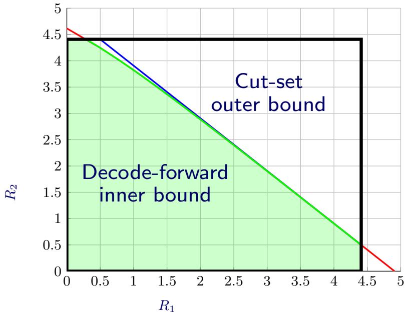
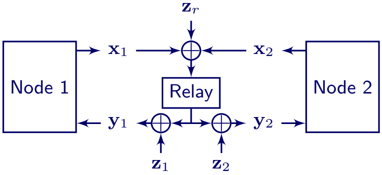


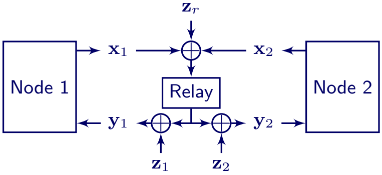


Sum-rate

Let us focus on the sum-rate
$R_\Sigma = R_1 + R_2$:



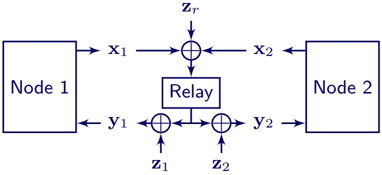
Sum-rate



Let us focus on the sum-rate $R_\Sigma = R_1 + R_2$:

- simplifying assumption: $P_1 = P_2 = P_r = P$, $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 1$,

Sum-rate



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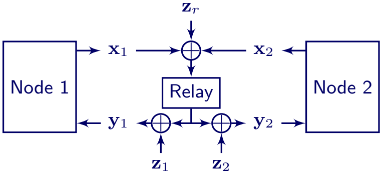

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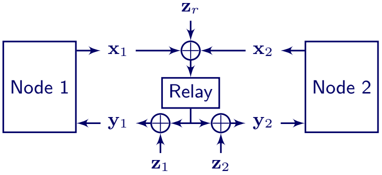
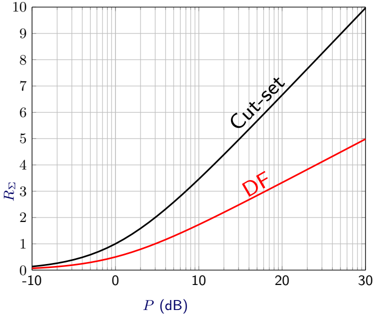
$\Rightarrow$ max $R_{DF} = \frac{1}{2}\log(1+P)$





Sum-rate

Let us focus on the sum-rate $R_\Sigma = R_1 + R_2$:

- simplifying assumption:
  $P_1 = P_2 = P_r = P$,
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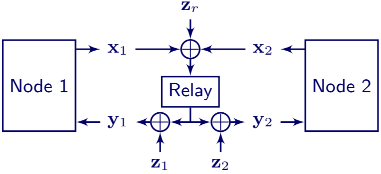
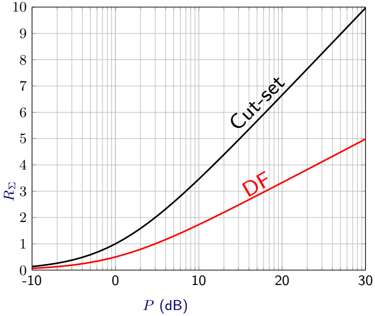
- BC: $R_\Sigma \le \frac{1}{2}\log(1+P)$,

$\Rightarrow$ max $R_{DF} = \frac{1}{2}\log(1+P)$

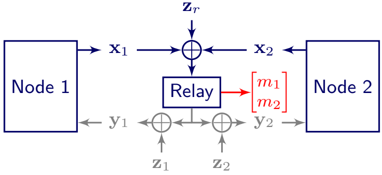
- Question: How to improve?





Network Coding

- Treat uplink as a MAC $\Rightarrow$ $R_\Sigma \leq \frac{1}{2}\log(1+2P)$.



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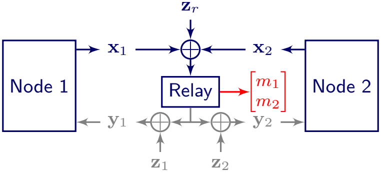

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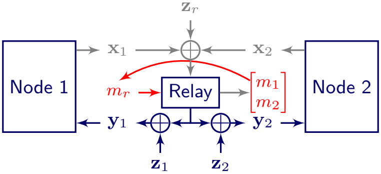

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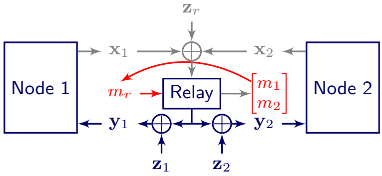

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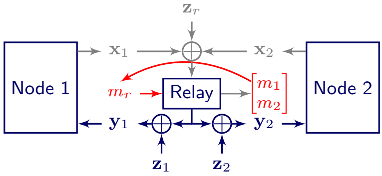

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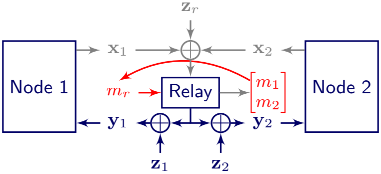

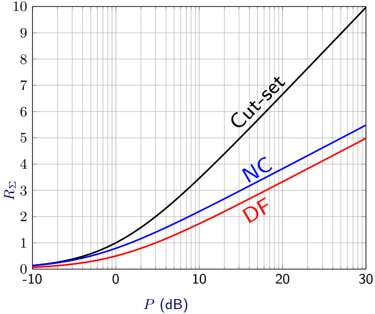

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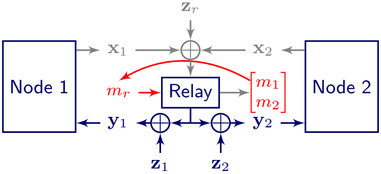

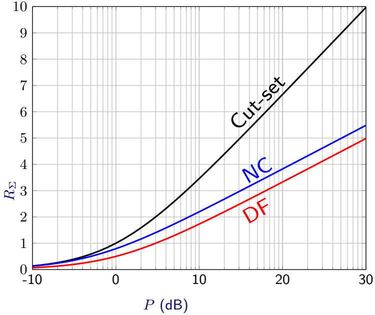

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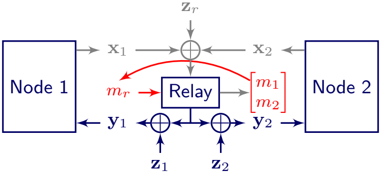
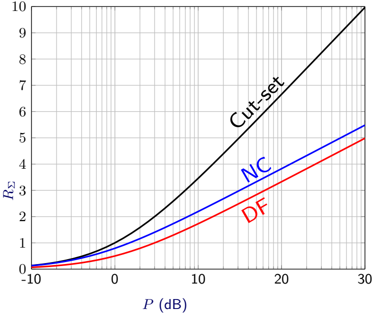
- Further improvement?





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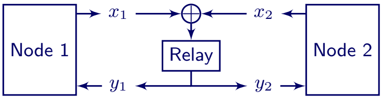
Important insight: Relay does not need to decode the messages!



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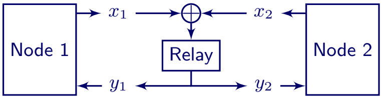

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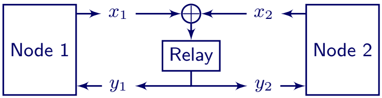

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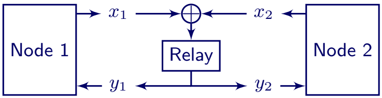

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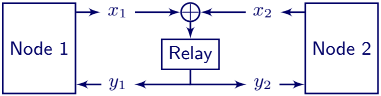

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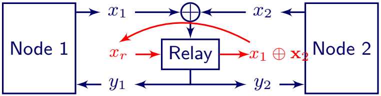

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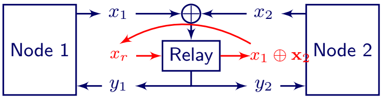

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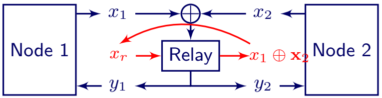

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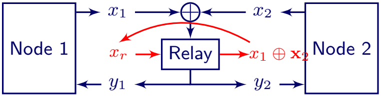

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[Avestimehr *et al.* 07]

Analyse using the Linear Deterministic (LD) Model

Outline

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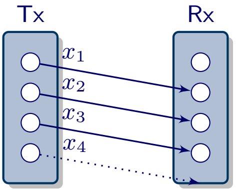

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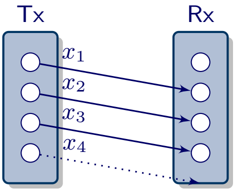

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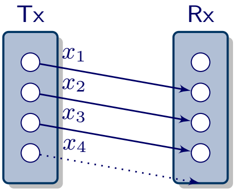

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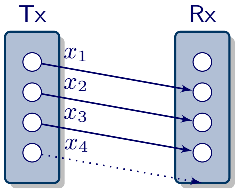

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Deterministic P2P

A Gaussian P2P can be approximated as a binary channel with input $\mathbf{x} = [x_1,\ x_2, \cdots, x_q]^T$ and output $\mathbf{y} = \mathbf{S}^{q-n}\mathbf{x}$ where $q \geq n = \left\lceil \frac{1}{2} \log(P) \right\rceil$ and $\mathbf{S} = \begin{bmatrix} \mathbf{0} & 0 \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$ is a down-ward shift matrix.

The linear-deterministic approximation

Similar approximation can be applied to the:

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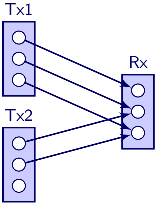
MAC with $n_1 \geq n_2$



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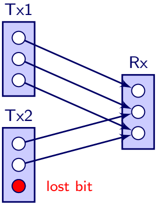

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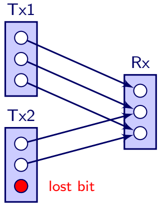

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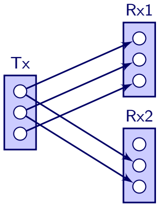

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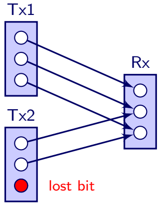

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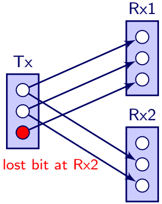

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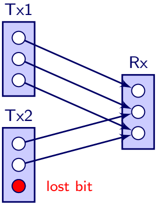
MAC with $n_1 \geq n_2$



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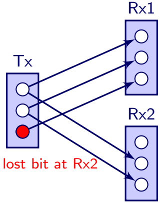

The linear-deterministic approximation

Similar approximation can be applied to the:

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- A very useful tool for studying Gaussian networks
- Obtained insights in an LD network can be extended to corresponding Gaussian networks

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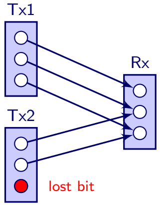

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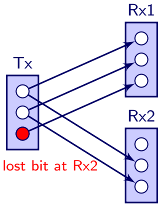
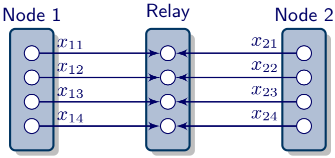


LD Two-way relay channel



- Uplink: MAC with $n_1 = n_2 = n = 4$,
- Node i sends $\mathbf{x}_i \in \mathbf{F}_2^q$,
- Relay receives $\mathbf{S}^{q-n}\mathbf{x}_1 \oplus \mathbf{S}^{q-n}\mathbf{x}_2$,
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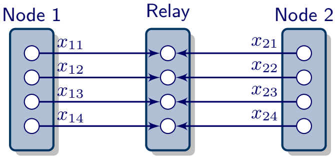

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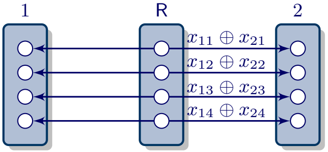

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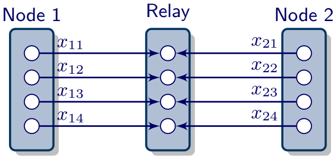

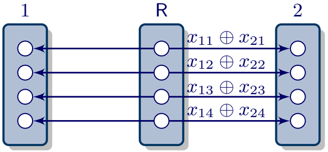

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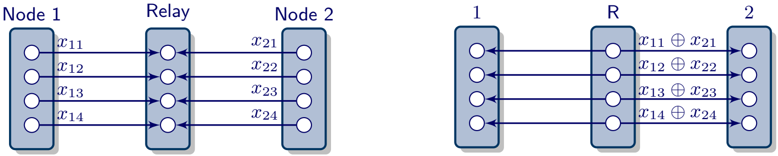
Compute-forward (a.k.a. Physical-layer NC)

Relay decodes a function (sum) of the transmit signals, and forwards this sum. Each node can decode the desired signal using its own signal as side information.

LD Two-way relay channel



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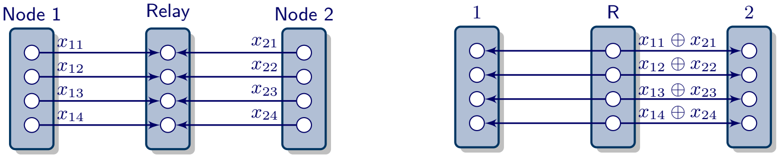

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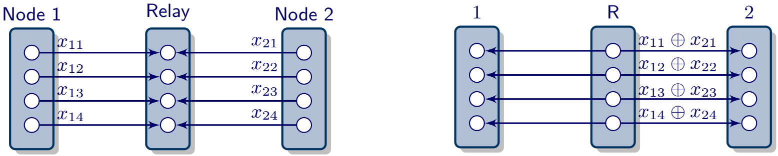

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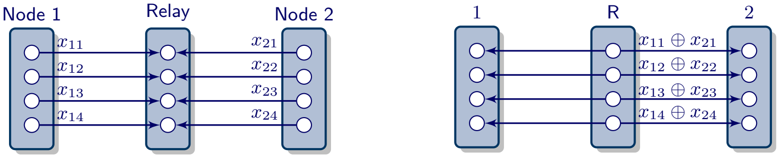

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CF (almost) doubles the rate in comparison to DF and to NC (at high SNR).

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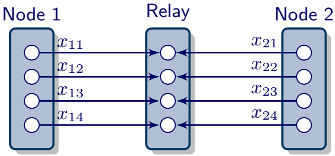

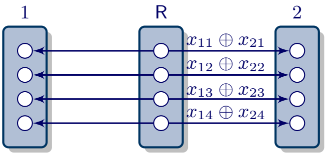

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How to extend to Gaussian two-way relay channels?

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- Now what?

Outline

1. Two-way channel

2. Two-way relay channel
   - The linear-deterministic approximation
   - Lattice codes

3. Multi-way relay channel
   - Multi-pair Two-way Relay Channel
   - Multi-way Relay Channel

4. Multi-way Channel

Computation

Definition (Computation)

Computation is the process of recovering a function of transmit codewords from a received sequence of symbols after sending the codewords through a channel.

How?

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How?

Computation can be accomplished by using lattice codes.

Idea: Codes located on a grid so that the sum of two codewords is a codeword.

Lattice-codes

Property: u_1 and u_2 lattice codes $\Rightarrow u_1 + u_2$ lattice code!

Examples:

- $\mathbb{Z}$ is a one-dimensional lattice

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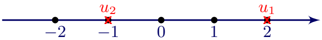

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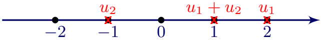

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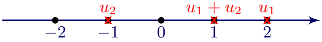

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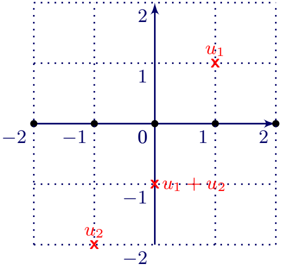

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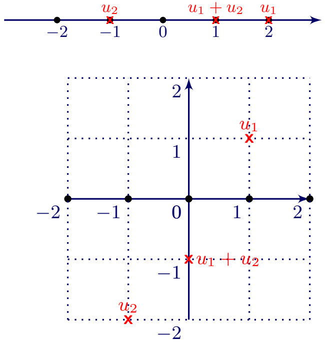

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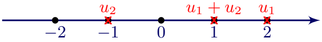

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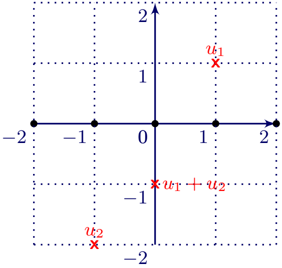

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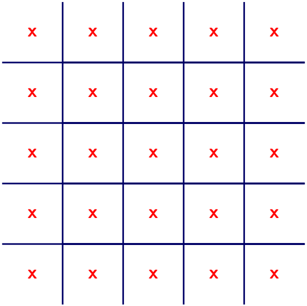
What is a nested lattice code?

Nested-lattice codes

- **x**: Fine lattice Λ_f



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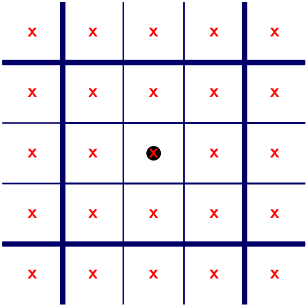

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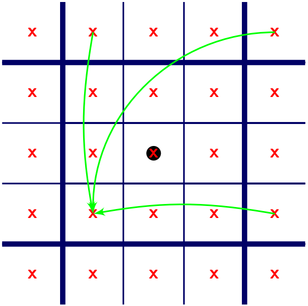

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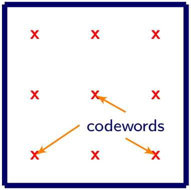

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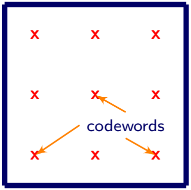

Back to the two-way relay channel

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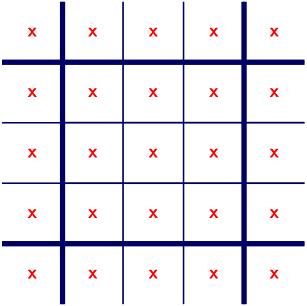

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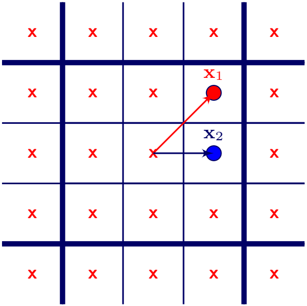

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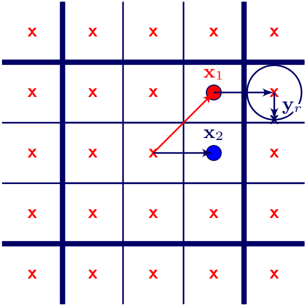

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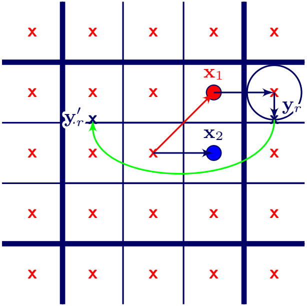

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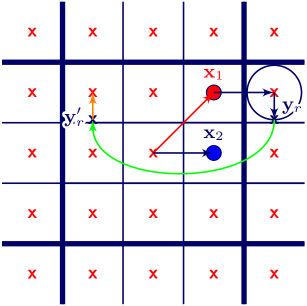

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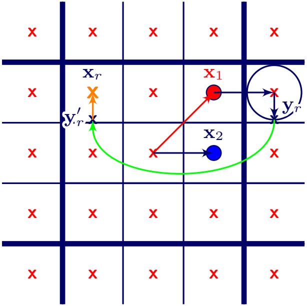

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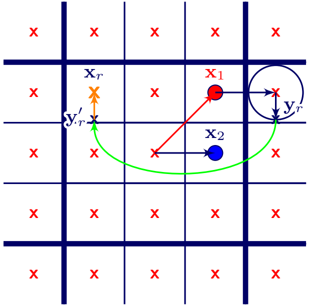

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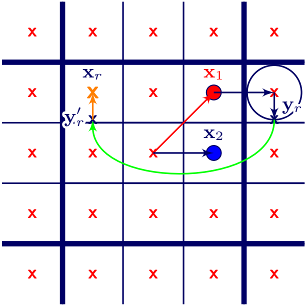

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Computation rate [Nazer & Gastpar 11]

Relay can compute $(\mathbf{x}_1 + \mathbf{x}_2) \bmod \Lambda_c$ as long as $R \leq \left[\frac{1}{2}\log\left(\frac{1}{2} + P\right)\right]^+$.

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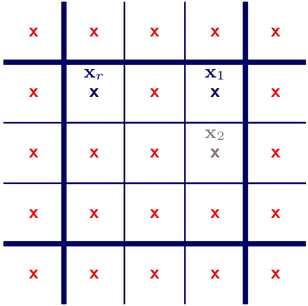

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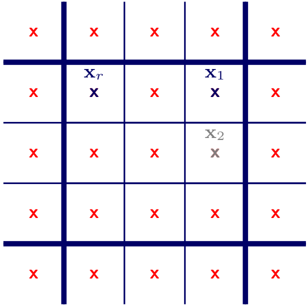

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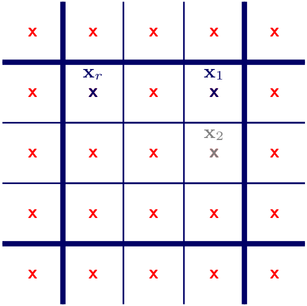
- node i decodes $\mathbf{x}_r \Rightarrow$ $R \leq \frac{1}{2}\log(1+P)$.
- Can $\mathbf{x}_j$ be recovered from $\mathbf{x}_r$?



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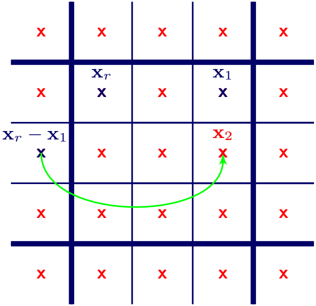

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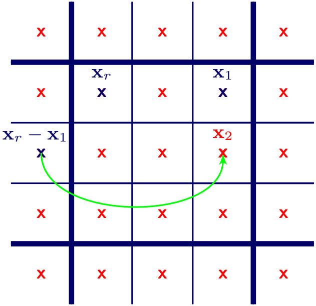
$$R \leq \left[\frac{1}{2}\log\left(\frac{1}{2}+P\right)\right]^+ \quad \text{uplink}$$

$$R \leq \frac{1}{2}\log\left(1+P\right) \quad \text{downlink}$$



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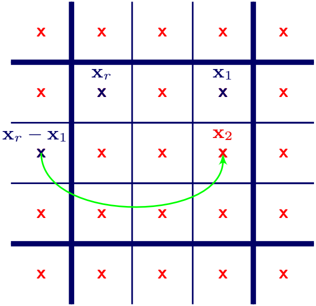


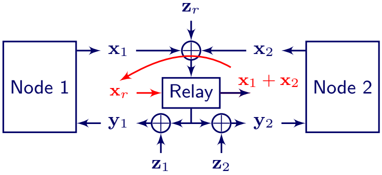
Max sum-rate for CF: $R_{CF} = \left[\log\left(\frac{1}{2}+P\right)\right]^+$

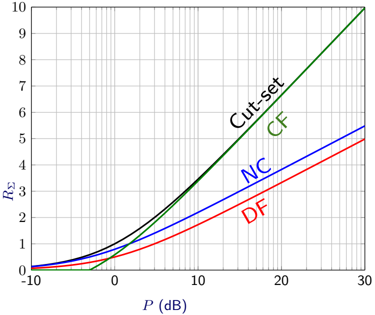
CF sum-rate

$$R_{DF} = \frac{1}{2}\log(1+P)$$

$$R_{NC} = \frac{1}{2}\log(1+2P)$$

$$R_{CF} = \left[\log\left(\frac{1}{2}+P\right)\right]^{+}.$$





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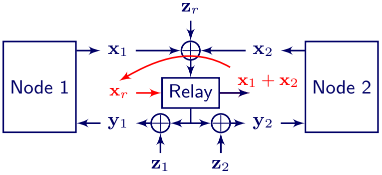
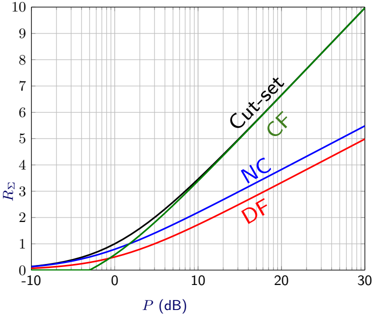
- CF doubles the rate (at high SNR),





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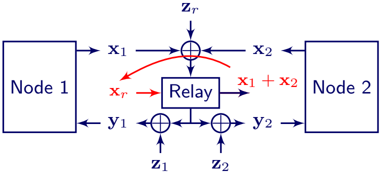
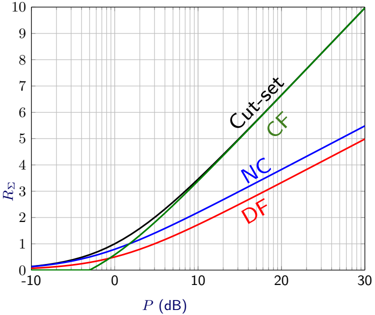
- CF doubles the rate (at high SNR),

+ Close to capacity at high SNR

- zero rate at low SNR





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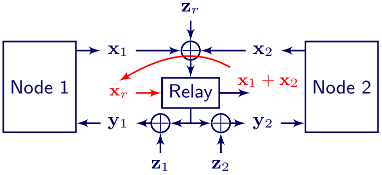
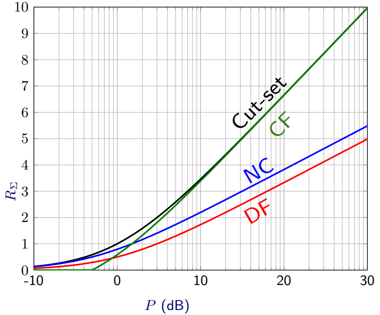
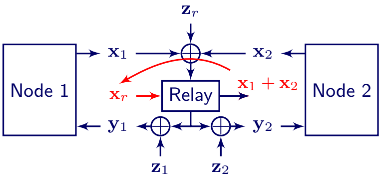
- CF doubles the rate (at high SNR),
- \+ Close to capacity at high SNR
- \- zero rate at low SNR
- Best scheme: combination of CF and NC.



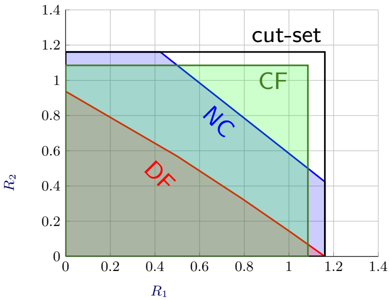


CF rate region



- CF achieves

$$R_1, R_2 \leq \left[\frac{1}{2}\log\left(\frac{1}{2}+P\right)\right]^+.$$

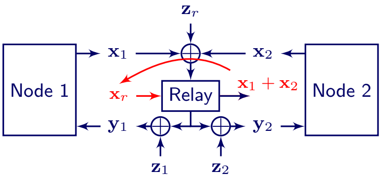


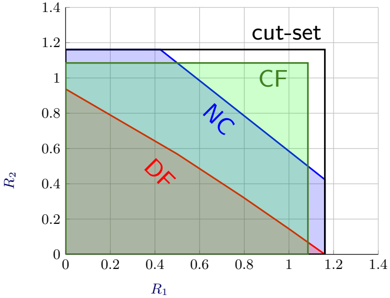
CF rate region

- CF achieves

$$R_1, R_2 \leq \left[\frac{1}{2}\log\left(\frac{1}{2}+P\right)\right]^+.$$

- highest sum-rate



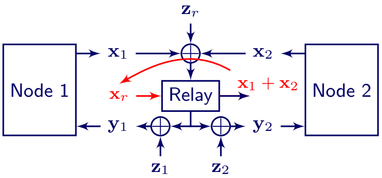


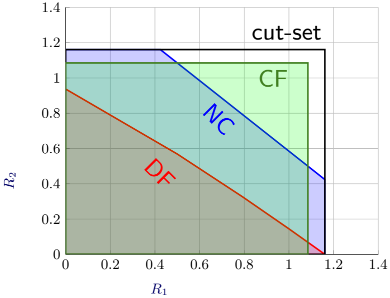
CF rate region

- CF achieves

$$R_1, R_2 \leq \left[\frac{1}{2}\log\left(\frac{1}{2}+P\right)\right]^+.$$

- highest sum-rate
- But: NC is better in some regions,





CF rate region

- CF achieves

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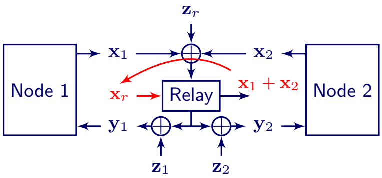
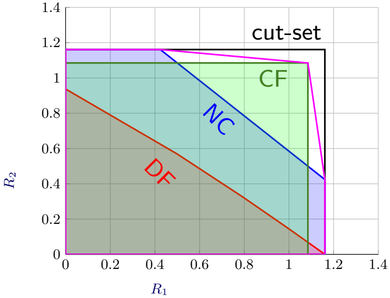
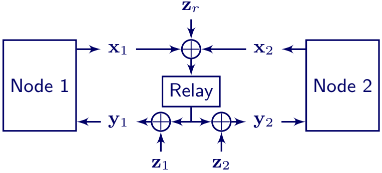
- highest sum-rate
- But: NC is better in some regions,
- Best: Time-sharing NC and CF,





CF rate region

What happens if $P_1 \geq P_2$ and P_r arbitrary?



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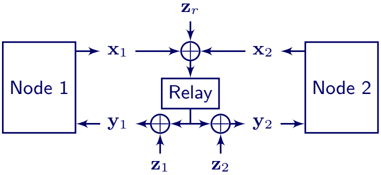

- Reduce P_1 to P_2 and use CF:

$$R_1, R_2 \leq \left[\frac{1}{2}\log\left(\frac{1}{2} + P_2\right)\right]^+$$

$$R_1, R_2 \leq \frac{1}{2}\log\left(1 + P_r\right)$$

CF rate region

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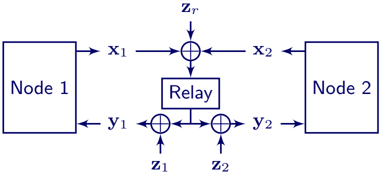

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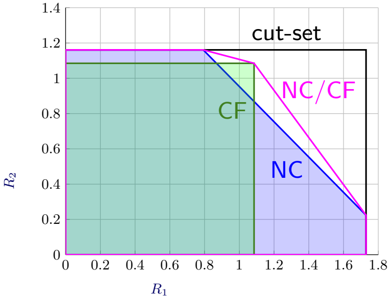

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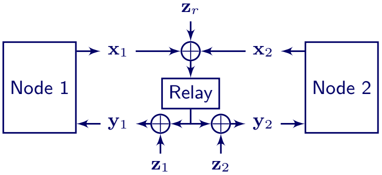

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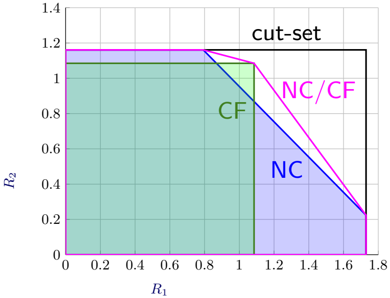
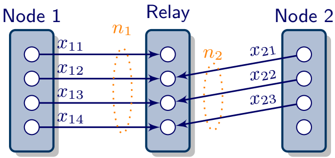
- Time-sharing NC and CF,
- Far from sum-capacity!
- Can we do better?

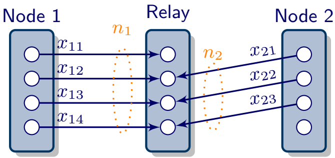


Back to LD



- $P_1 \geq P_2 \Rightarrow n_1 \geq n_2$,
- use n_2 bits for CF,
- use $n_1 - n_2$ bits for DF,
- R_1 and R_2 achievable if $R_1 \leq n_1$ and $R_2 \leq n_2$

Back to LD



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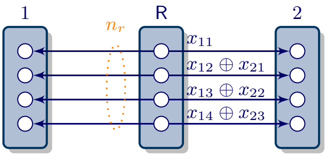
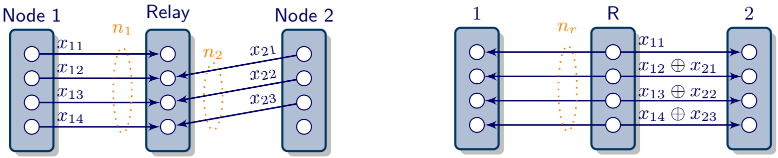


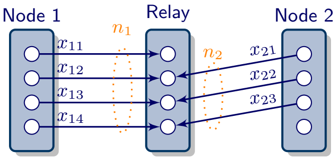
- n_r bit-pipes,
- node 2 gets $\mathbf{x}_1$,
- node 1 gets $\mathbf{x}_2$,
- R_1 and R_2 achievable if $\max\{R_1, R_2\} \leq n_r$,

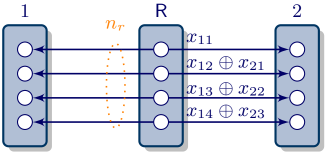
LD Two-way relay channel



- Achievable rates: $R_1 \leq \min\{n_1, n_r\}$ and $R_2 \leq \min\{n_2, n_r\}$.

LD Two-way relay channel

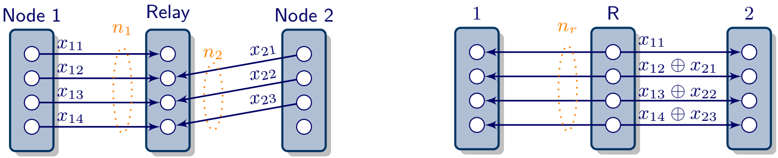




- Achievable rates: $R_1 \leq \min\{n_1, n_r\}$ and $R_2 \leq \min\{n_2, n_r\}$.

$\Rightarrow$ $R_1 \leq \lceil \frac{1}{2} \log(\min\{P_1, P_r\}) \rceil$, $R_2 \leq \lceil \frac{1}{2} \log(\min\{P_2, P_r\}) \rceil$,

LD Two-way relay channel



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- asymmetric rates can be achieved by combining CF and DF

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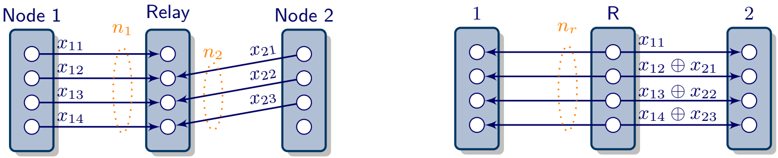

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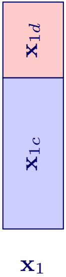
- asymmetric rates can be achieved by combining CF and DF

How to extend to Gaussian two-way relay channels?

CF rate region

Combination of CF and DF

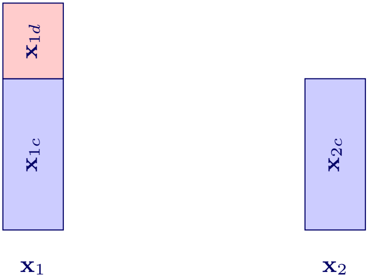
- Node 1: $\mathbf{x}_1 = \sqrt{P_d}\mathbf{x}_{1d} + \sqrt{P_c}x_{1c}$



CF rate region

Combination of CF and DF

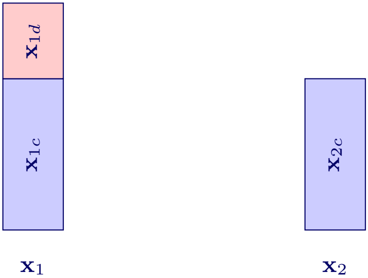
- Node 1: $\mathbf{x}_1 = \sqrt{P_d}\mathbf{x}_{1d} + \sqrt{P_c}x_{1c}$
- Node 2: $\mathbf{x}_2 = \sqrt{P_c}\mathbf{x}_{2c}$



CF rate region

Combination of CF and DF

- Node 1: $\mathbf{x}_1 = \sqrt{P_d}\mathbf{x}_{1d} + \sqrt{P_c}x_{1c}$
- Node 2: $\mathbf{x}_2 = \sqrt{P_c}\mathbf{x}_{2c}$
- Powers: $P_c + P_d \leq P_1$, $P_c \leq P_2$.

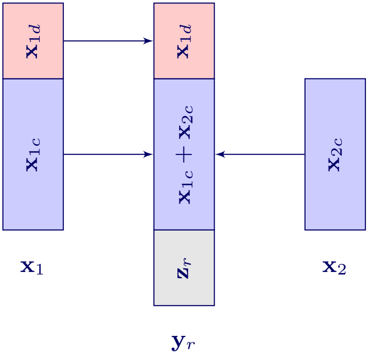


CF rate region

Combination of CF and DF

- Node 1: $\mathbf{x}_1 = \sqrt{P_d}\mathbf{x}_{1d} + \sqrt{P_c}x_{1c}$
- Node 2: $\mathbf{x}_2 = \sqrt{P_c}\mathbf{x}_{2c}$
- Powers: $P_c + P_d \leq P_1$, $P_c \leq P_2$.
- Relay receives:

$$\mathbf{y}_r = \sqrt{P_d}\mathbf{x}_{1d} + \sqrt{P_c}(x_{1c} + x_{2c}) + \mathbf{z}_r,$$



CF rate region

Combination of CF and DF

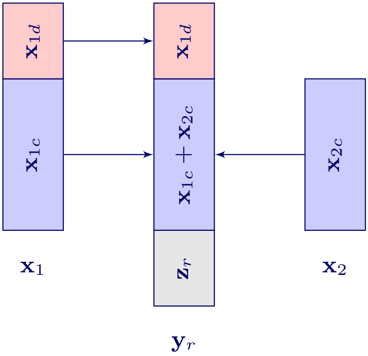
- Node 1: $\mathbf{x}_1 = \sqrt{P_d}\mathbf{x}_{1d} + \sqrt{P_c}x_{1c}$
- Node 2: $\mathbf{x}_2 = \sqrt{P_c}\mathbf{x}_{2c}$
- Powers: $P_c + P_d \leq P_1,\ P_c \leq P_2.$
- Relay receives:

$$\mathbf{y}_r = \sqrt{P_d}\mathbf{x}_{1d} + \sqrt{P_c}(x_{1c} + x_{2c}) + \mathbf{z}_r,$$

- decodes $\mathbf{x}_{rd} = \mathbf{x}_{1d}$ followed by $\mathbf{x}_{rc} = \mathbf{x}_{1c} + \mathbf{x}_{2c}$,

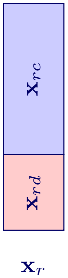
$$R_d \leq \frac{1}{2}\log\left(1 + \frac{P_d}{1 + 2P_c}\right)$$

$$R_c \leq \left[\frac{1}{2}\log\left(\frac{1}{2} + P_c\right)\right]^+$$



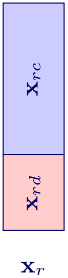
CF rate region

- $\mathbf{x}_r = \sqrt{P_{rc}}\mathbf{x}_{rc} + \sqrt{P_{rd}}\mathbf{x}_{rd},$



CF rate region

- $\mathbf{x}_r = \sqrt{P_{rc}}\mathbf{x}_{rc} + \sqrt{P_{rd}}\mathbf{x}_{rd}$,
- Powers: $P_{rc} + P_{rd} \leq P_r$,



CF rate region

- $\mathbf{x}_r = \sqrt{P_{rc}}\mathbf{x}_{rc} + \sqrt{P_{rd}}\mathbf{x}_{rd}$,
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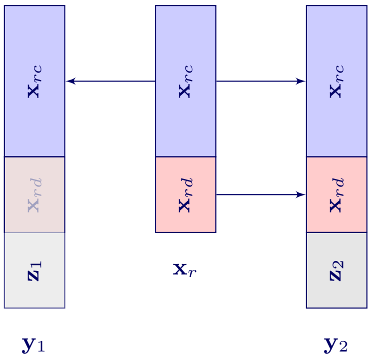
- Node i receives
  $\mathbf{y}_i = \sqrt{P_{rc}}\mathbf{x}_{rc} + \sqrt{P_{rd}}\mathbf{x}_{rd} + \mathbf{z}_i$



CF rate region

- $\mathbf{x}_r = \sqrt{P_{rc}}\mathbf{x}_{rc} + \sqrt{P_{rd}}\mathbf{x}_{rd}$,
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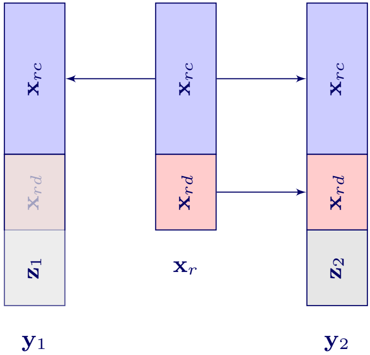
- Node i receives
  $\mathbf{y}_i = \sqrt{P_{rc}}\mathbf{x}_{rc} + \sqrt{P_{rd}}\mathbf{x}_{rd} + \mathbf{z}_i$
- both nodes decode $\mathbf{x}_{rc}$, and extract desired CF signal



CF rate region

- $\mathbf{x}_r = \sqrt{P_{rc}}\mathbf{x}_{rc} + \sqrt{P_{rd}}\mathbf{x}_{rd}$,
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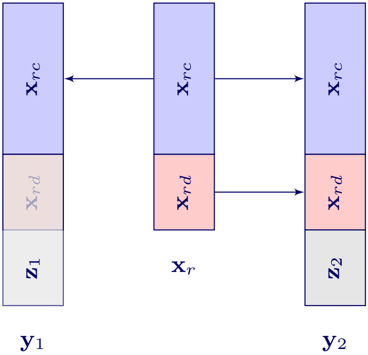
- Node i receives
  $\mathbf{y}_i = \sqrt{P_{rc}}\mathbf{x}_{rc} + \sqrt{P_{rd}}\mathbf{x}_{rd} + \mathbf{z}_i$
- both nodes decode $\mathbf{x}_{rc}$, and extract desired CF signal
- and node 2 decodes $\mathbf{x}_{rd}$

$$R_c \leq \frac{1}{2}\log\left(1 + \frac{P_{rc}}{1 + P_{rd}}\right)$$

$$R_d \leq \frac{1}{2}\log\left(1 + P_{rd}\right)$$



CF/DF

Combining CF and DF achieves $R_1 = R_c + R_d$, $R_2 = R_c$, where

$$R_d \leq \min\left\{\frac{1}{2}\log\left(1+\frac{P_d}{1+2P_c}\right), \frac{1}{2}\log\left(1+P_d\right)\right\}$$

$$R_c \leq \min\left\{\left[\frac{1}{2}\log\left(\frac{1}{2}+P_c\right)\right]^+, \frac{1}{2}\log\left(1+\frac{P_c}{1+P_d}\right)\right\}$$

for $P_c + P_d \leq P_1$, $P_c \leq P_2$, $P_{rc} + P_{rd} \leq P_r$.

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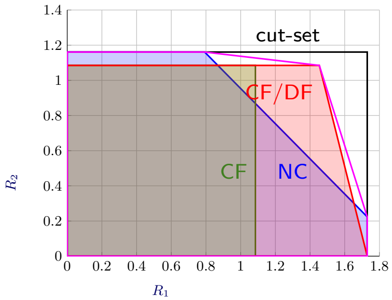
Achieves capacity within a constant gap

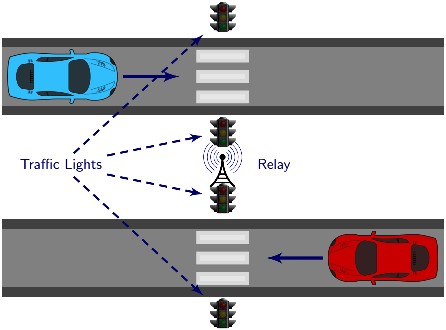


Summary

- Key ingredient: CF using lattice codes (physical-layer network coding)
- Best scheme: Combination of CF, DF, and NC,
- Sum-capacity scales as $\log(P)$, (optimal scaling)

Summary

- Key ingredient: CF using lattice codes (physical-layer network coding)
- Best scheme: Combination of CF, DF, and NC,
- Sum-capacity scales as $\log(P)$, (optimal scaling)
- Consequence: Using a relay as a two-way relay doubles the rate of communication, which is of interest for applications with a delay constraint

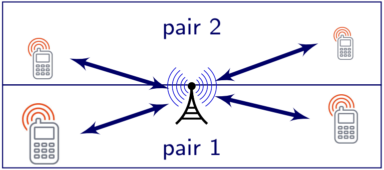


Outline

Multi-pair Two-way Relay Channel

Multiple users communicating pair-wise through a relay [S. *et al.* 09]

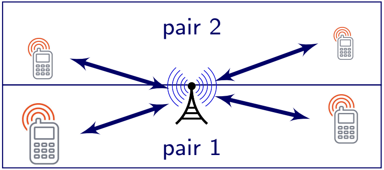
- Combination of CF and DF,



Multi-pair Two-way Relay Channel

Multiple users communicating pair-wise through a relay [S. *et al.* 09]

- Combination of CF and DF,
- In each pair (i_k, j_k), one node sends $\mathbf{x}_{i_k d} + \mathbf{x}_{i_k c}$ and the other $\mathbf{x}_{j_k c}$,



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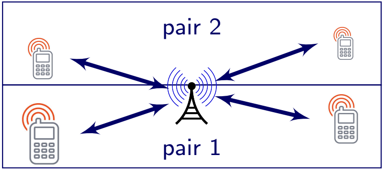
- Relay decodes $\mathbf{x}_{i_k d}$ then $\mathbf{x}_{i_k c} + \mathbf{x}_{j_k c}$ of pair k, then pair k'...

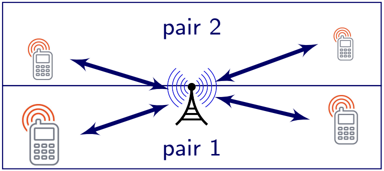


$$R_{kd} \leq \frac{1}{2}\log\left(1 + \frac{P_{kd}}{1 + 2P_{kc} + \sum_{\ell=k+1}^{K}(P_{\ell d} + 2P_{\ell c})}\right)$$

$$R_{kc} \leq \left[\frac{1}{2}\log\left(\frac{1}{2} + \frac{P_{kc}}{1 + \sum_{\ell=k+1}^{K}(P_{\ell d} + 2P_{\ell c})}\right)\right]^{+}$$

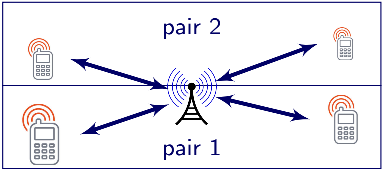
Multi-pair Two-way Relay Channel

- Relay forwards a scaled sum of the decoded signals



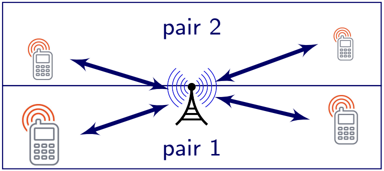
Multi-pair Two-way Relay Channel

- Relay forwards a scaled sum of the decoded signals
- Nodes in pair k decode the signals successively, starting with pair 1 ending with pair k



Multi-pair Two-way Relay Channel

- Relay forwards a scaled sum of the decoded signals
- Nodes in pair k decode the signals successively, starting with pair 1 ending with pair k
- within a constant of the cut-set bound in the Gaussian case,



Remarks

The multi-pair case is similar to the single pair case:

- Sum-rate scaling of $\log(P)$, (optimal scaling)
- Cut-set bounds are nearly tight. Achievability requires:
- CF: Bi-directional communication between two nodes via the relay, and
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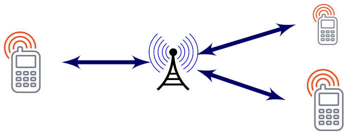
Do we require new ingredients in multi-user cases?

Outline

Multi-way Relay Channel

Channel with multiple users communicating in all directions via a relay [Lee & Lim 09]



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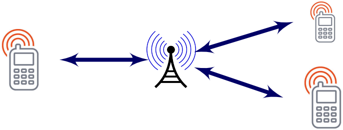

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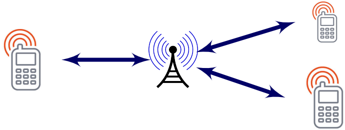

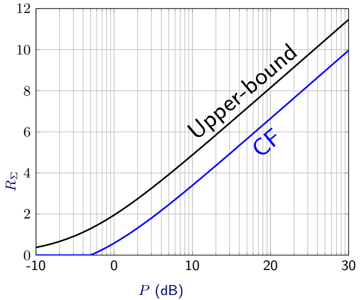

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- Cut-set bound scaling of $\frac{3}{2}\log(P)$
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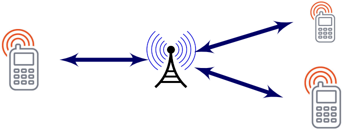

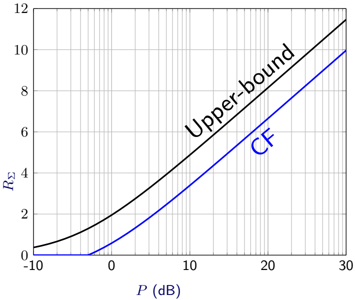

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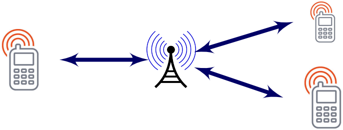
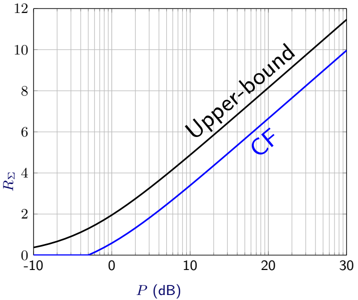
- CF achieves the optimal scaling





Multi-way Relay Channel

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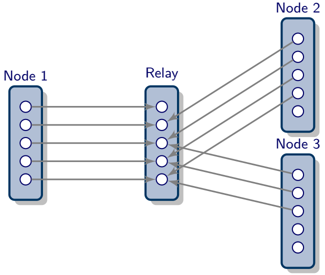
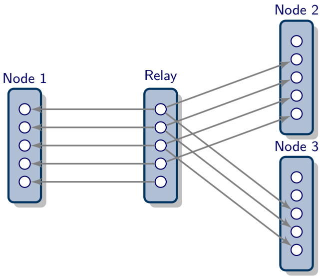
- Multi-way: Similar?
- No!
- Let us check the LD model

Multi-way Relay Channel

Outer bound: Cut-set and genie-aided.

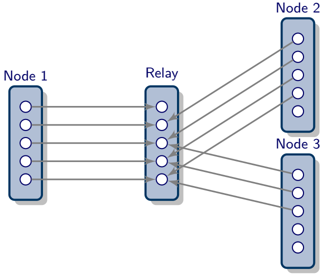


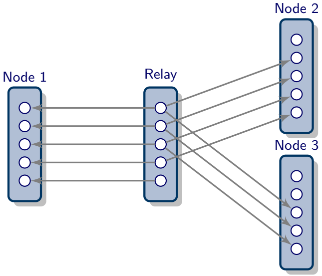


Multi-way Relay Channel

Outer bound: Cut-set and genie-aided.

- Let $n_1 = 5$, $n_2 = 4$, $n_3 = 3$,

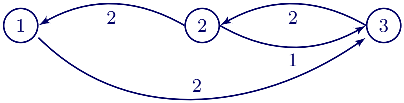


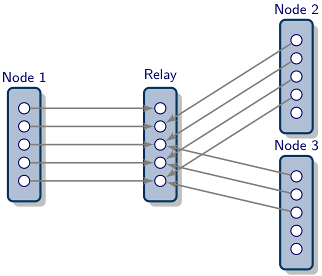


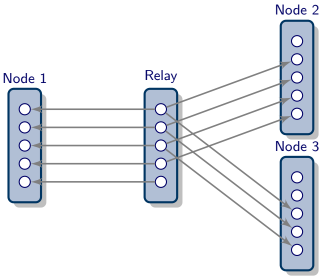
Multi-way Relay Channel

Outer bound: Cut-set and genie-aided.

- Let $n_1 = 5$, $n_2 = 4$, $n_3 = 3$,
- $R_{13} = R_{21} = R_{32} = 2$, $R_{23} = 1$







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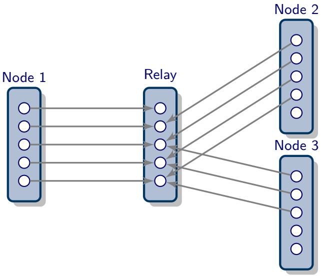

Outer bound: Cut-set and genie-aided.

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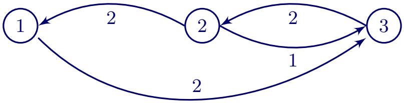

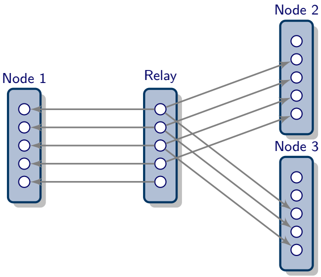

- Rates inside the outer bound

Multi-way Relay Channel

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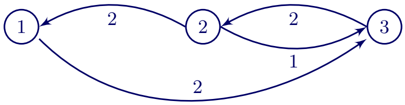

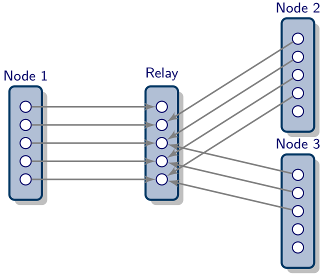

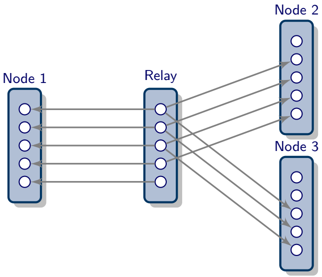

- Rates inside the outer bound
- Achievable?

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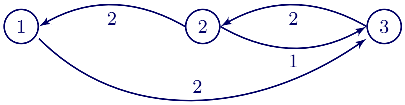
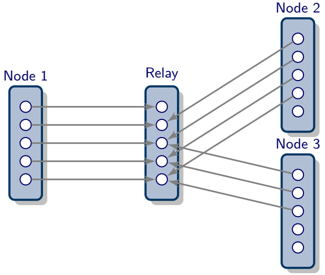
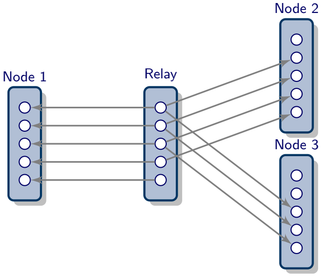
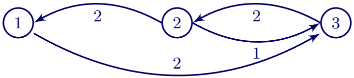




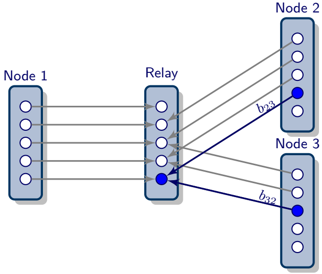
- Rates inside the outer bound
- Achievable?
- Try bi-directional and uni-directional

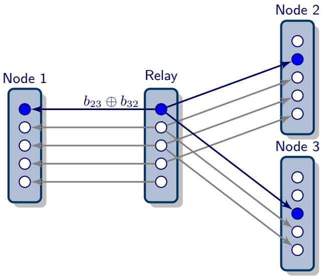


Multi-way Relay Channel



- Bi-directional $2 \leftrightarrow 3$





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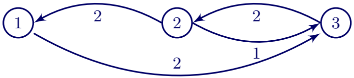

- Bi-directional $2 \leftrightarrow 3$
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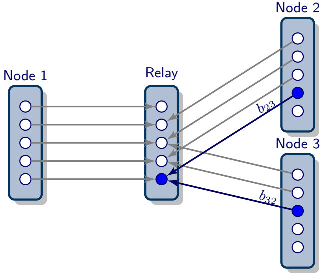

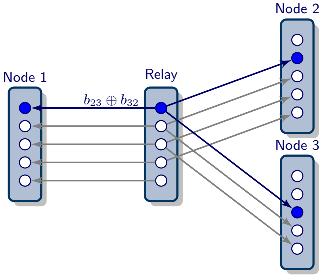

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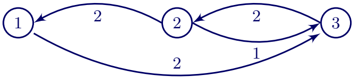

- Bi-directional $2 \leftrightarrow 3$
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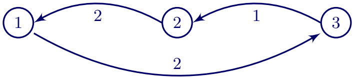

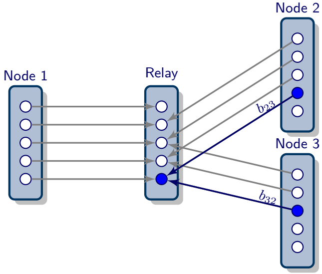

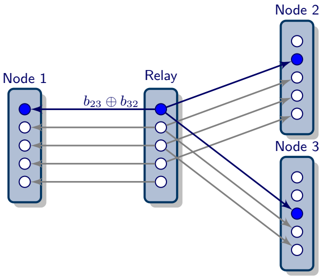

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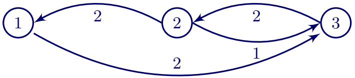

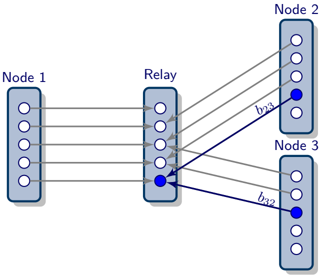

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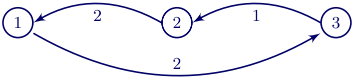
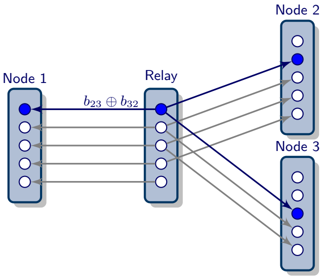


- Uni-directional $1 \rightarrow 3$, $2 \rightarrow 1$, and $3 \rightarrow 2$ requires 5 bit-pipes



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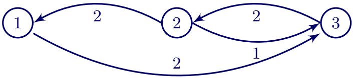

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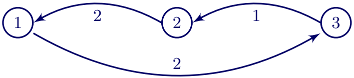
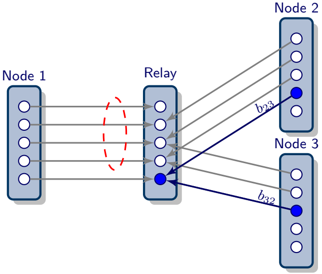
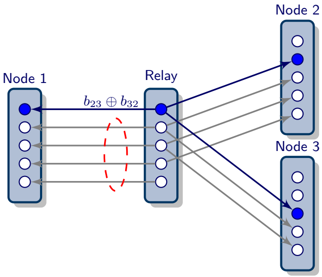


- Uni-directional $1 \rightarrow 3$, $2 \rightarrow 1$, and $3 \rightarrow 2$ requires 5 bit-pipes
- Relay has only 4 remaining!





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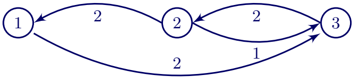
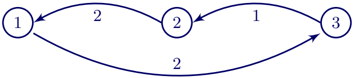
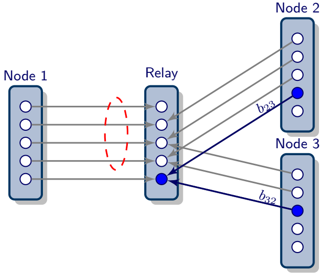


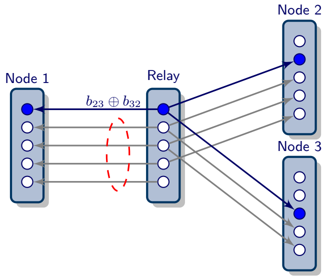
- Bi-directional $2 \leftrightarrow 3$
- Achieves $r_{23} = r_{32} = 1$
- Remainder $R_{13} = R_{21} = 2$, $R_{32} = 1 \Rightarrow$



- Uni-directional $1 \to 3$, $2 \to 1$, and $3 \to 2$ requires 5 bit-pipes
- Relay has only 4 remaining!

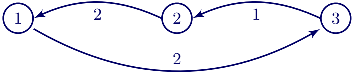
$\Rightarrow$ Achievability requires more CF

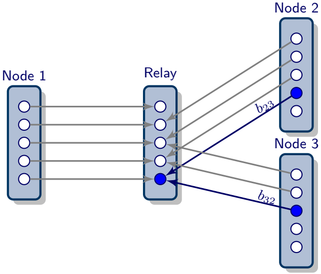


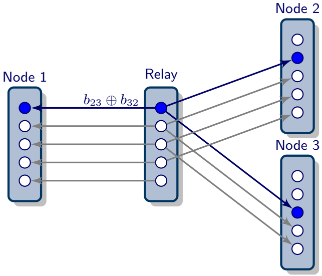


Multi-way Relay Channel

- Bi-directional $2 \leftrightarrow 3$







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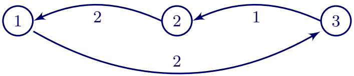
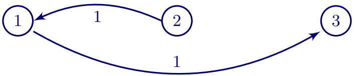
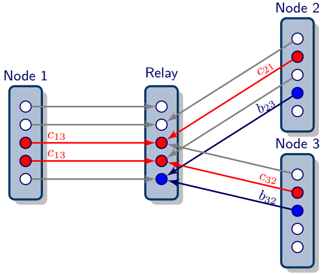
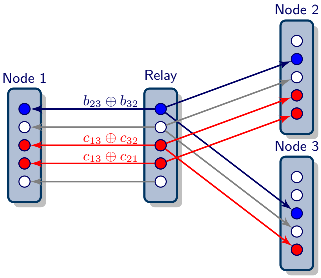


- Cyclic $1 \rightarrow 3 \rightarrow 2 \rightarrow 1$:
- Achieves $r_{13} = r_{32} = r_{21} = 1$
- Remainder $R_{13} = R_{21} = 1$







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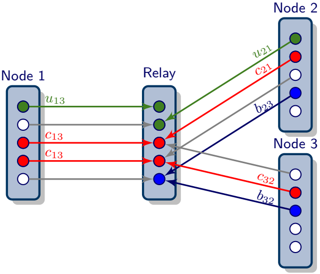

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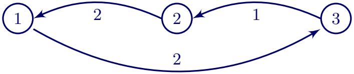

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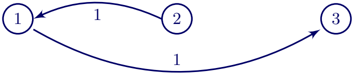
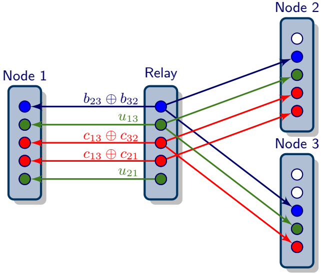
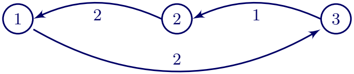


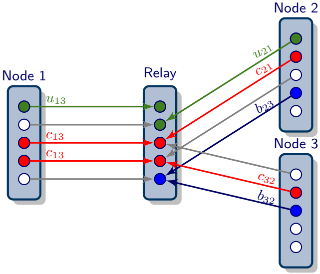


- Uni-directional $1 \to 3$ and $2 \to 1$: requires 2 bit-pipes
- Relay has 2 remaining

Multi-way Relay Channel

- Bi-directional $2 \leftrightarrow 3$





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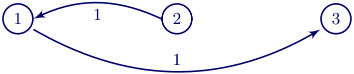
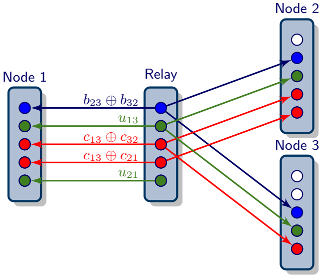
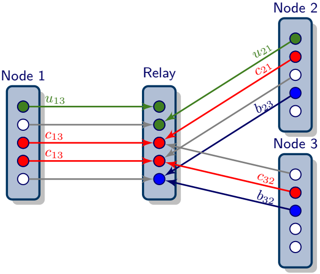


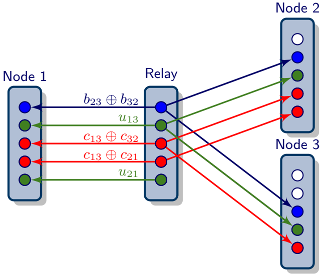


- Uni-directional $1 \to 3$ and $2 \to 1$: requires 2 bit-pipes
- Relay has 2 remaining
- Desired rate achieved!

Multi-way Relay Channel

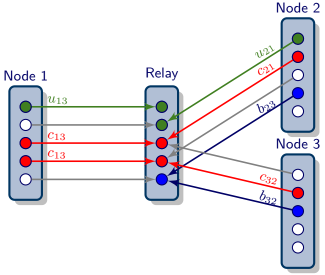
- Additional ingredient: Cyclic Communication

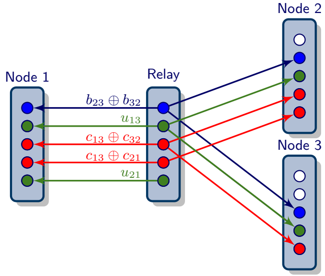




Multi-way Relay Channel

- Additional ingredient: Cyclic Communication
- Remark:
  Bi-directional: 2 bits per bit-pipe
  Cyclic: 3/2 bits per bit-pipe
  Uni-directional: 1 bit per bit-pipe





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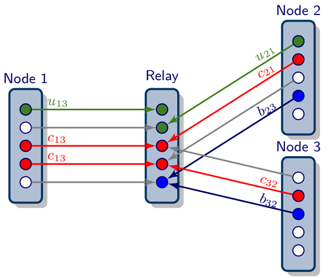
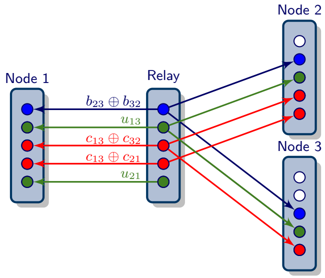
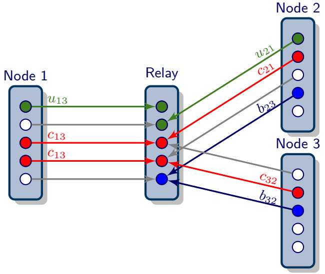




Multi-way Relay Channel



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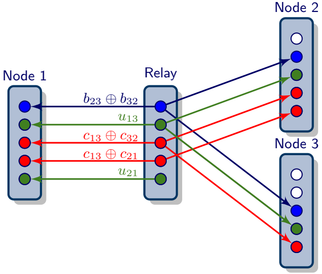
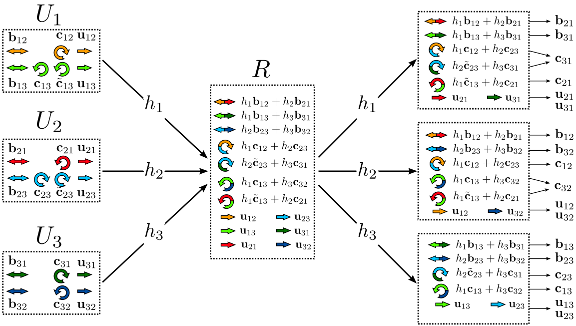
- LD case: Capacity achieving [C. & S. 11]



Multi-way Relay Channel

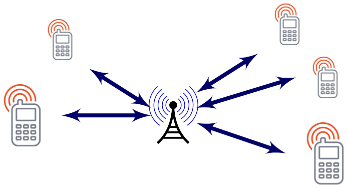


Gaussian case:

- node i sends $\mathbf{x}_{ib} + \mathbf{x}_{ic} + \mathbf{x}_{iu}$ (bi-directional, cyclic, uni-directional)
- relay computes the sum of bi-directional signals, cyclic signals, and decodes the uni-directional ones
- nodes decode successively and obtain their desired signals
- Problem reduces to power allocation (near optimal allocation in [C. & S. 12])

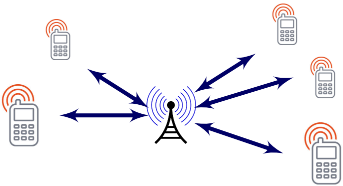
K-user case

- Sum-capacity upper bound scales as $\log(P)$,



K-user case

- Sum-capacity upper bound scales as $\log(P)$,
- Simple scheduling: Schedule one pair of users at a time
- Channel reduces to a sequence of two-way relay channels



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- Sum-capacity upper bound scales as $\log(P)$,
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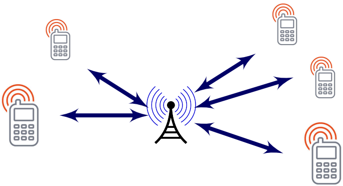
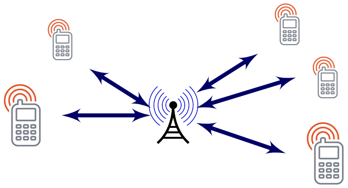
- Apply bi-directional communication over each two-way relay channel



K-user case

- Sum-capacity upper bound scales as $\log(P)$,
- Simple scheduling: Schedule one pair of users at a time
- Channel reduces to a sequence of two-way relay channels
- Apply bi-directional communication over each two-way relay channel
- achieves sum-capacity within a constant gap

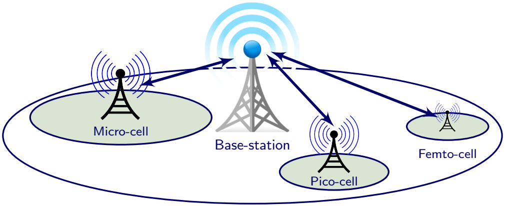


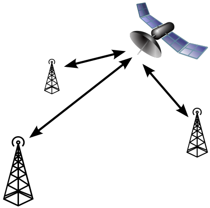
Summary

- Key ingredient: CF for bi-directional and cyclic communication
- Best scheme: Combination of bi-directional, cyclic, and uni-directional
- Sum-capacity scales as $\log(P)$,

Summary

- Key ingredient: CF for bi-directional and cyclic communication
- Best scheme: Combination of bi-directional, cyclic, and uni-directional
- Sum-capacity scales as $\log(P)$,
- Consequence: Treating different modes of information flow differently increases the communication rate



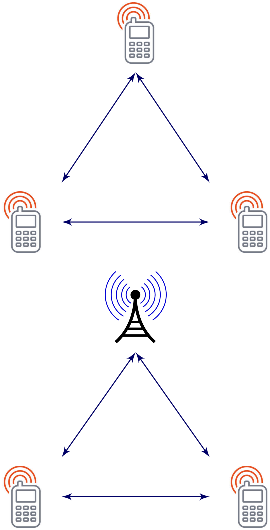


Outline

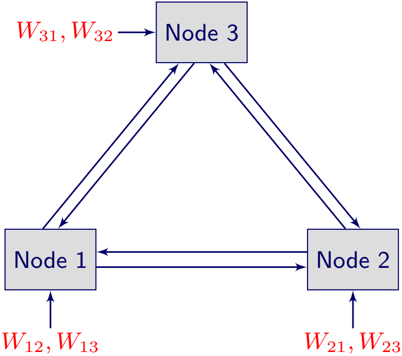
1. Two-way channel
2. Two-way relay channel
   - The linear-deterministic approximation
   - Lattice codes
3. Multi-way relay channel
   - Multi-pair Two-way Relay Channel
   - Multi-way Relay Channel
4. **Multi-way Channel**

3-Way Channel

- 3 (or more) nodes communicating with each other in multiple directions
- Extension of Shannon's two-way channel
- A suitable model for D2D systems (offloading traffic from the cellular network [Asadi *et al.*])

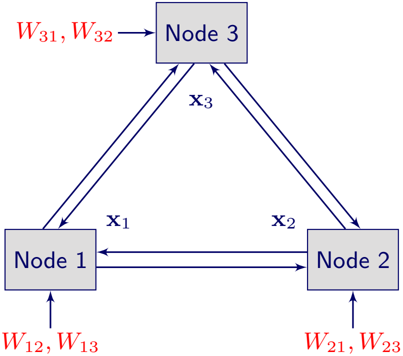


3-Way Channel (3WC)



- Full message-exchange: Message W_{ij} from node i to j,

3-Way Channel (3WC)



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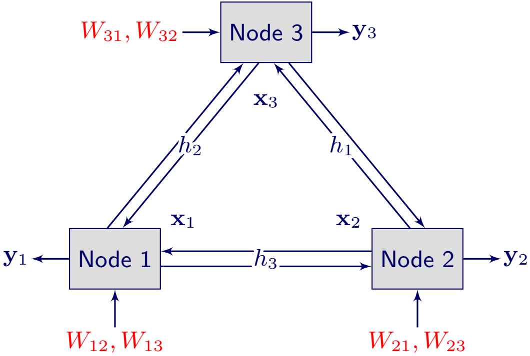

- Full message-exchange: Message W_{ij} from node i to j,
- Tx signal: $\mathbf{x}_j$, power P,
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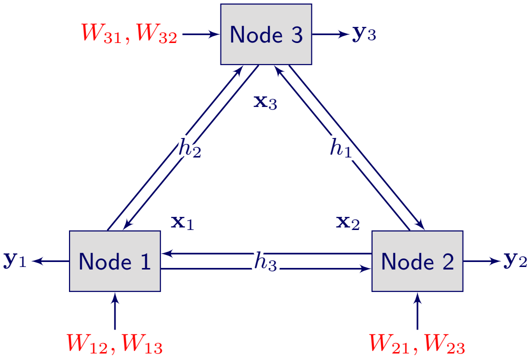

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- w.l.o.g. $h_3^2 \geq h_2^2 \geq h_1^2$,

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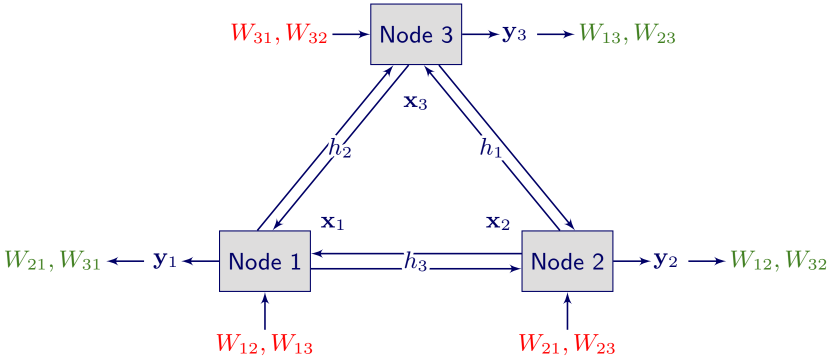

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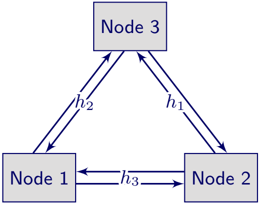
- w.l.o.g. $h_3^2 \geq h_2^2 \geq h_1^2$,
- Node k decodes W_{ik} and W_{jk},

Sum-Capacity

- Two-way channel: Cut-set bound tight, capacity scales as $\log(P)$,
- 3-way channel: Cut-set bound not tight, capacity also scales as $\log(P)$



Sum-capacity

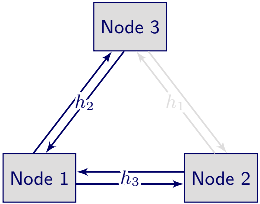
The sum-capacity of the 3-way channel is bounded by

$$\log(1 + h_3^2 P) \leq C_\Sigma \leq \log(1 + h_3^2 P) + 2.$$

- Converse: Genie-aided bound [C. *et al.* 14]
- Achievability: Only users 1 and 2 communicate
- Optimal scaling can also be achieved by scheduling

Capacity Region

- Two-way channel scheme suffices for sum-capacity,



Capacity Region

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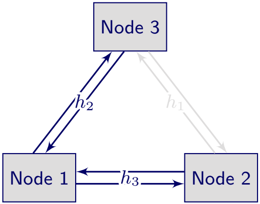
- but not for Capacity region,



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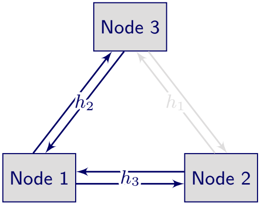

- Assume nodes 2 & 3 want to communicate, but $h_1^2 \ll 1$

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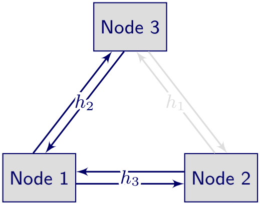

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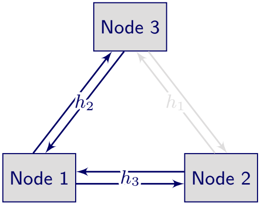
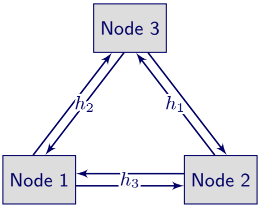


- Assume nodes 2 & 3 want to communicate, but $h_1^2 \ll 1$
- Communication still possible via node 1 as a relay (two-way relay channel)
- Relaying is necessary for capacity region!

Capacity Region

How to find the capacity region?

- Trick: Transform the channel into a Y-channel!



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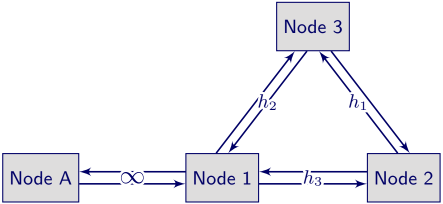
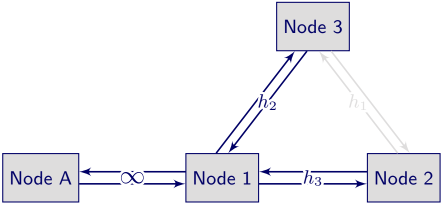
- Split stronger node into two,



Capacity Region

How to find the capacity region?

- Trick: Transform the channel into a Y-channel!
- Split stronger node into two,
  - Assume $h_1^2 = 0 \Rightarrow$ Y-channel!



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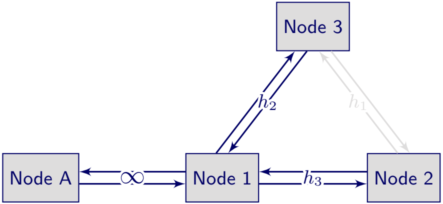

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$\Rightarrow$ Capacity achieving scheme for the Y-channel is capacity achieving for the 3-way channel

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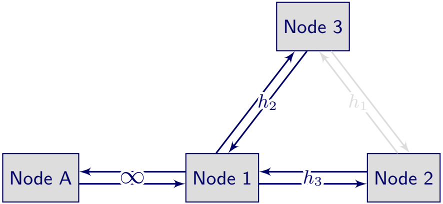


- Assume $h_1^2 = 0 \Rightarrow$ Y-channel!

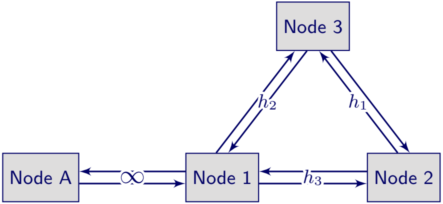
$\Rightarrow$ Capacity achieving scheme for the Y-channel is capacity achieving for the 3-way channel

- What if $h_1^2 > 0$?

Capacity Region

If $h_1^2 > 0$:

- Interference between nodes 2 and 3 (w.r.t. Y-channel scheme)



Capacity Region

If $h_1^2 > 0$:

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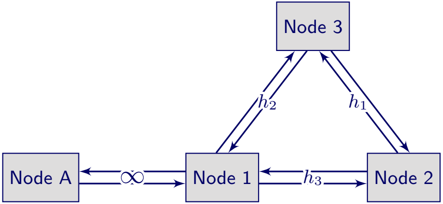
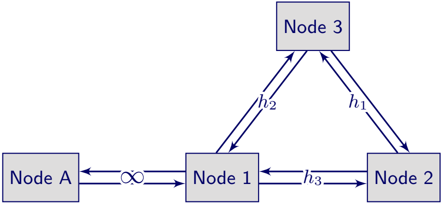
- How to resolve interference?



Capacity Region



If $h_1^2 > 0$:

- Interference between nodes 2 and 3 (w.r.t. Y-channel scheme)
- How to resolve interference?

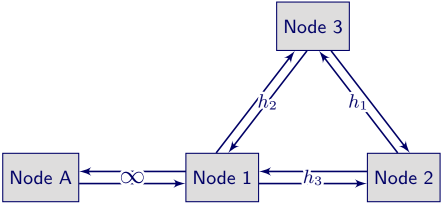
If interference at 3 is:

- A desired signal at 3: Backward decoding:

$$\mathbf{y}_3(B) = h_2\mathbf{x}_1(B) + h_1\mathbf{x}_{23}(B) + \mathbf{z}_3(B), \quad \mathbf{y}_3(B+1) = h_2\mathbf{x}_1(B+1) + \mathbf{z}_3(B+1)$$

- After decoding desired signals from $\mathbf{x}_1(B+1)$, node 3 removes interference from $\mathbf{x}_{23}(B)$ (Causality)

Capacity Region



If $h_1^2 > 0$:

- Interference between nodes 2 and 3 (w.r.t. Y-channel scheme)
- How to resolve interference?

If interference at 3 is:

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- After decoding desired signals from $\mathbf{x}_1(B+1)$, node 3 removes interference from $\mathbf{x}_{23}(B)$ (Causality)
- A desired signal at 1: Interference neutralization

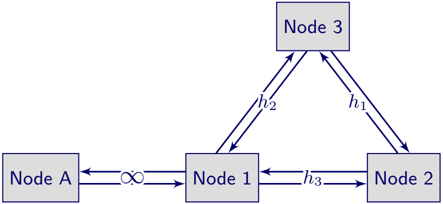
$$\mathbf{y}_3 = h_2\mathbf{x}_1 + h_1\mathbf{x}_{21} + \mathbf{z}_3$$

- Node 2 pre-transmits a signal for interference neutralization: $\mathbf{x}_1 = \mathbf{x}_1' - \frac{h_1}{h_2}\mathbf{x}_{21}$

Capacity Region

Main ingredients

- Y-channel scheme: Bi-directional, cyclic, and uni-directional communication



Capacity Region

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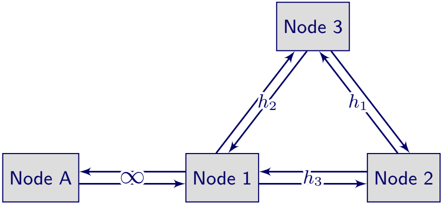

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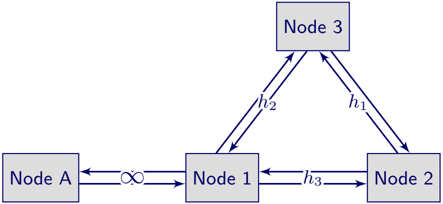

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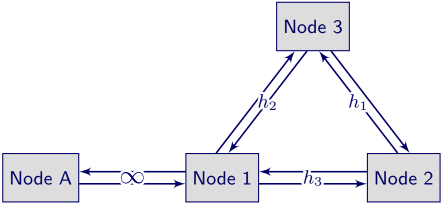
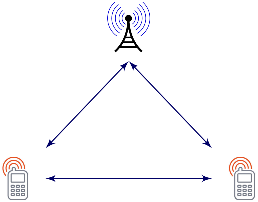


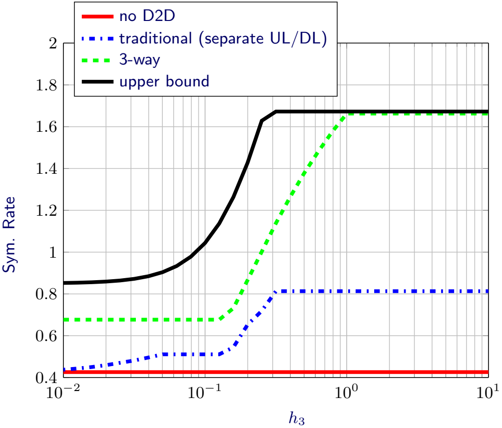
- Outer bound: Genie-aided and cut-set
- Capacity region of the LD case, and approximate capacity of the Gaussian case [C *et al.* 14],

Application

D2D communications:

Impact of (D2D) channel





Part 3: MIMO

Outline

From Capacity to Capacity Region

Single-user (MIMO P2P):

Input covariance $\mathbf{Q}$, tr$(\mathbf{Q}) \leq P$

From Capacity to Capacity Region

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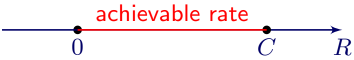

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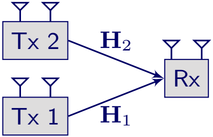

Input covariance $\mathbf{Q}_i$, $i = 1, 2$,
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From Capacity to Capacity Region

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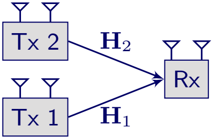

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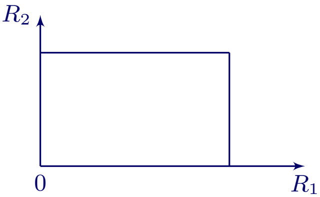

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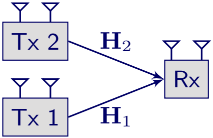

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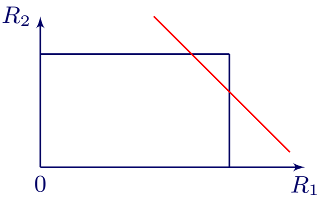

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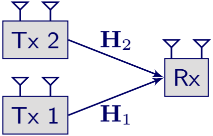

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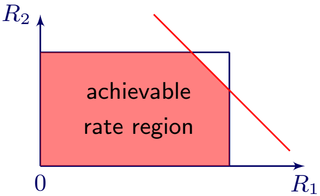

Capacity to DoF

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$C(P)$: SISO P2P capacity

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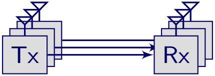

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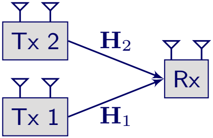

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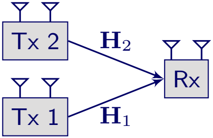

Capacity region:

$$R_i \leq \log |\mathbf{I} + \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^H|$$

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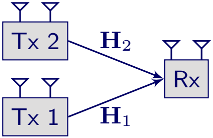

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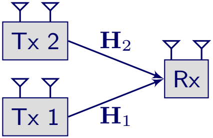

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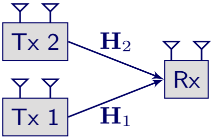

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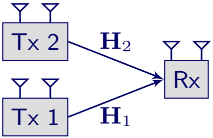

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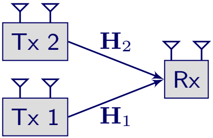

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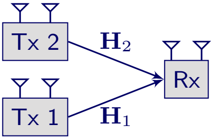

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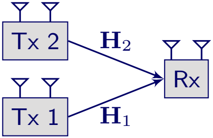

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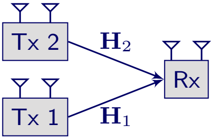

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⇒ $d_\Sigma = \min\{M_1 + M_2, N\}$

⇒ Sum-capacity equivalent to that of d_Σ parallel SISO P2P channels!

DoF

DoF

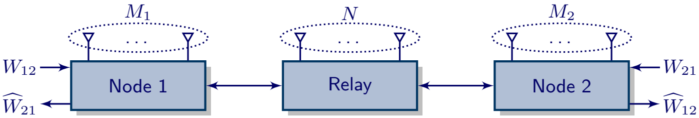
DoF d can be interpreted as the number of parallel streams that can be sent simultaneously over a channel.
It leads to a capacity approximation as

$$C_\Sigma = dC(P) + o\left(C(P)\right),$$

($C(P)$: capacity of a P2P channel)

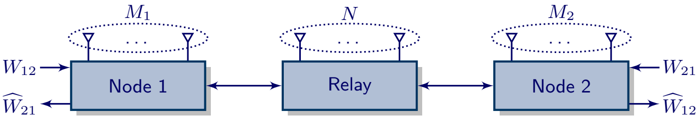
Outline

MIMO Two-Way Relay Channel



DoF characterization [Gündüz *et al.* 08]

MIMO Two-Way Relay Channel



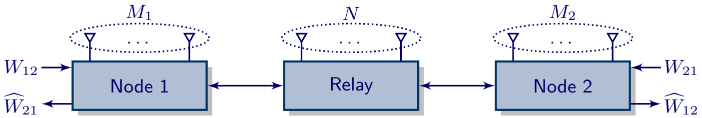
Cut-set bound:

- Node i can not send more than M_i streams,
- Node i can not receive more than M_i streams,
- Relay can relay at most $2N$ streams (PLNC gain),

$\Rightarrow$ Total streams $2\min\{M_1, M_2, N\}$ DoF

- achievable by Compress-forward e.g. [Gündüz *et al.* 08]

MIMO Two-Way Relay Channel



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- Next: Simple achievability scheme

Simple Achievability

Main Ingredients:

- Channel diagonalization
- Signal alignment

Outline

1. From Capacity to DoF
2. MIMO Two-Way Relay Channel
   - Channel diagonalization
   - Signal Alignment
3. MIMO multi-way relay channel
   - Sum-DoF
   - DoF Region
4. MIMO Multi-way Channel

Channel Diagonalization

Definition (Channel diagonlization)

Transform an arbitrary MIMO channel matrix $\mathbf{H}$ to a diagonal matrix.

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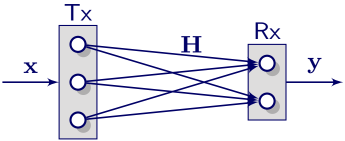
Transform an arbitrary MIMO channel matrix $\mathbf{H}$ to a diagonal matrix.

MIMO $M \times N$ P2P channel can be diagonalized by zero-forcing (ZF)

$M \geq N$: ZF pre-coding:



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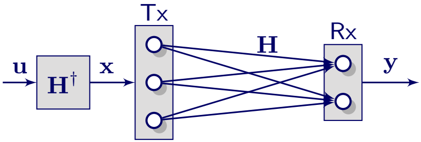

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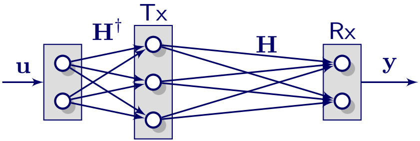

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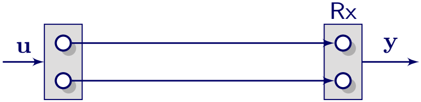

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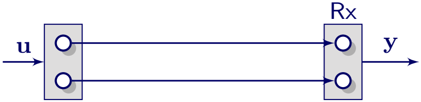

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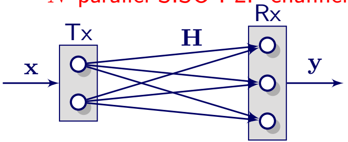

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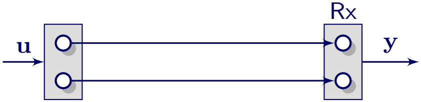

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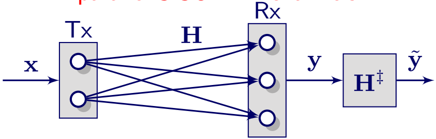

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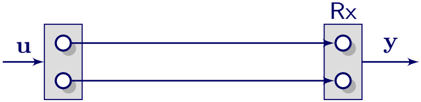

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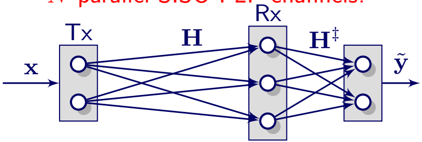

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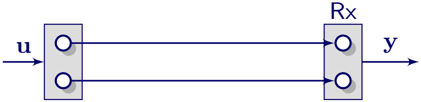

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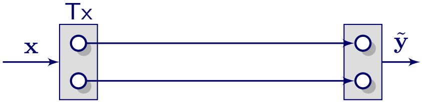

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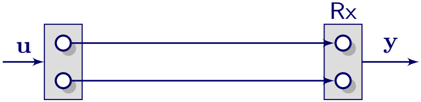

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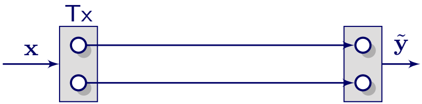

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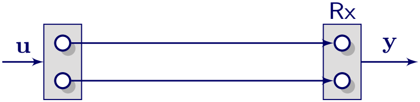

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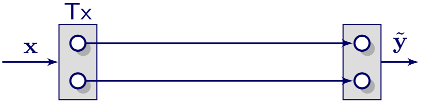

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MAC and BC are also separable

Outline

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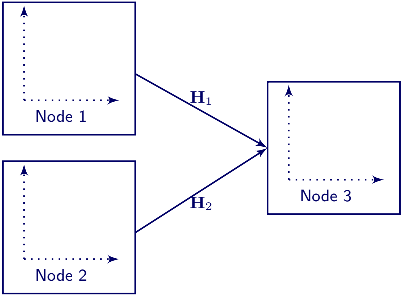

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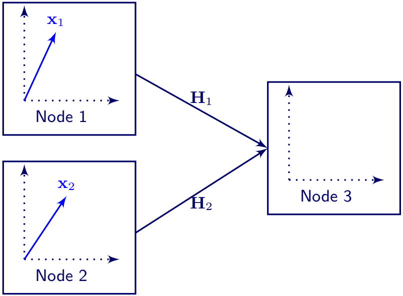

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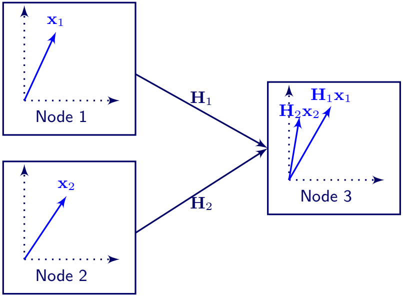

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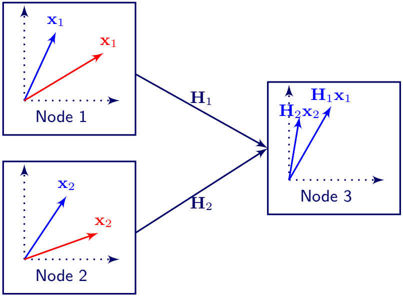

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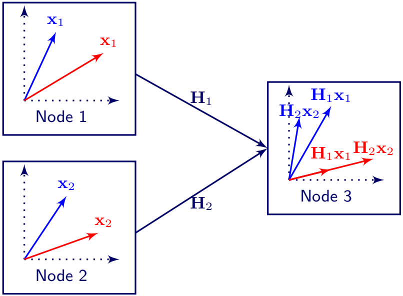

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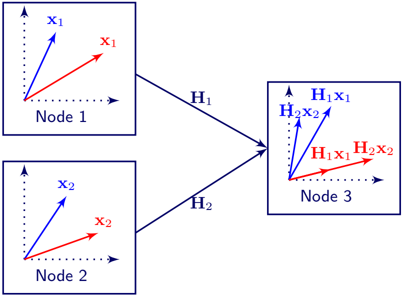
- Useful for CF!



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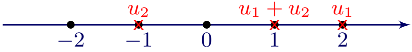

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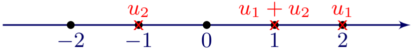

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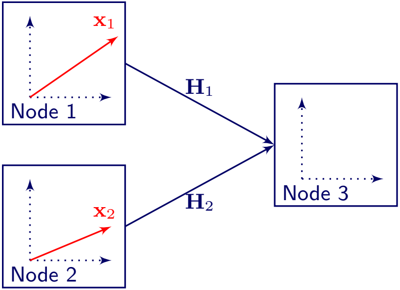

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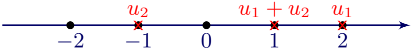

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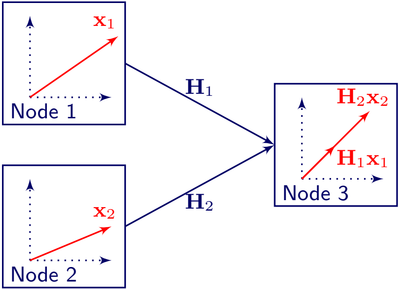

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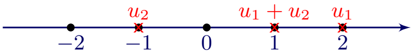

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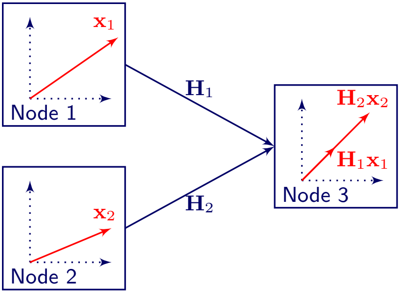
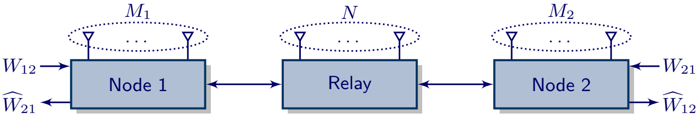
- compute $u_1 + u_2$ from $[1,\ 1]\mathbf{y}_3 = 2(u_1 + u_2) + n'$



Back to the MIMO Two-way Relay Channel



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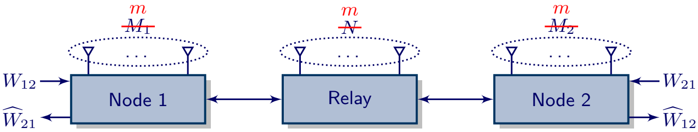

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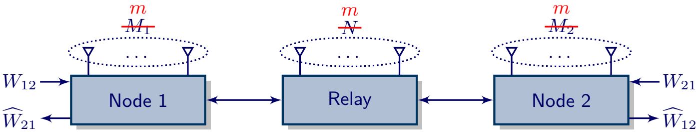

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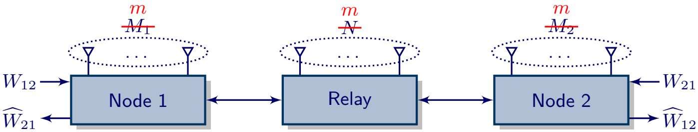

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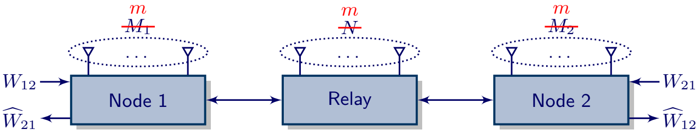

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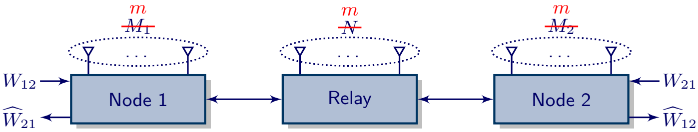

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$\Rightarrow$ $\mathbf{y}_r = \mathbf{u}_1 + \mathbf{u}_2 + \mathbf{z}_r, \quad \mathbf{y}_i' = \mathbf{x}_r + \mathbf{z}_i',$

$\Rightarrow$ m parallel SISO two-way relay channels

- Apply CF: achieve $2\left[\frac{1}{2}\log\left(\frac{1}{2}+P\right)\right]^+ \approx 2C(P)$ (at high P) per sub-channel
- Total rate $2mC(P) \Rightarrow 2m\ (= d)$ DoF

Remarks

- Diagonalization ensures the alignment of each pair of signals in a 1-D space

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- Diagonalization ensures the alignment of each pair of signals in a 1-D space
- Optimal DoF achievable by using either compress-forward, compute-forward, or amplify-forward over each sub-channel (dimension)

Possible improvement

DoF achieving scheme:

- Reduce the number of antennas to $\min\{M_1, M_2, N\}$,
- apply MIMO pre-coding and post-coding for channel diagonalization,

$\Rightarrow$ decompose channel into sub-channels,

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Use excess antennas for sending a (uni-directional) DF signal Dirty-paper coded against the remaining signals at the same node.

Outline

1. From Capacity to DoF
2. MIMO Two-Way Relay Channel
   - Channel diagonalization
   - Signal Alignment
3. MIMO multi-way relay channel
   - Sum-DoF
   - DoF Region
4. MIMO Multi-way Channel

MIMO Multi-way Relay Channel

MIMO Y-channel [Lee & Lim 09]

- inputs $\mathbf{x}_i$ and $\mathbf{x}_r$,
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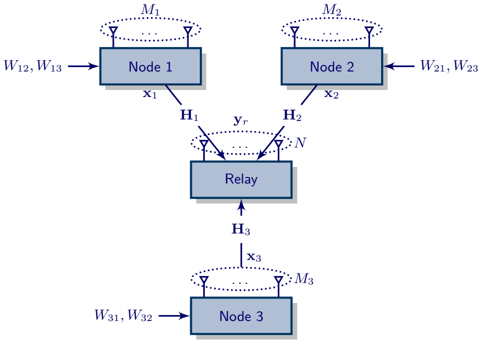

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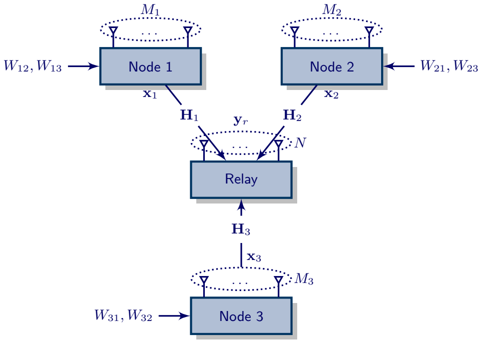

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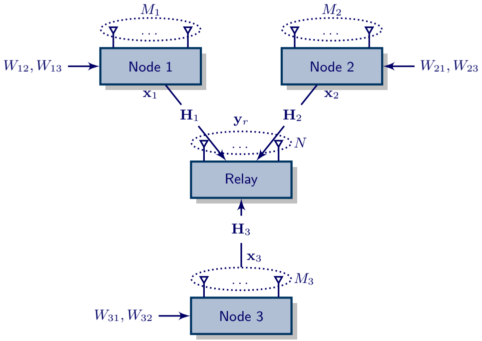

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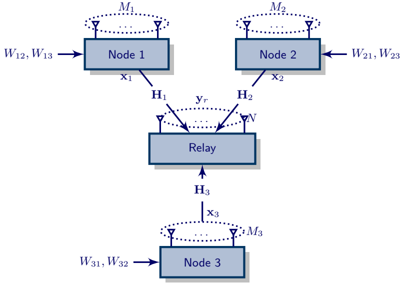

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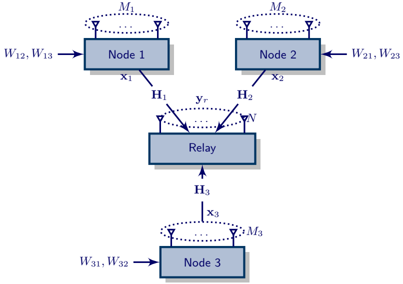

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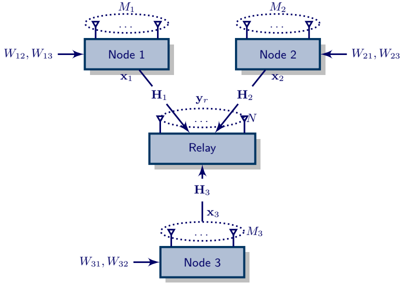


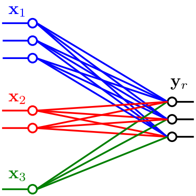
sum-DoF

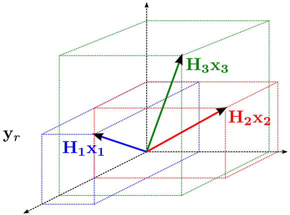
The sum-DoF of the MIMO Y-channel with $M_1 \geq M_2 \geq M_3$ (wlog) is given by

$$d = \min\{\underbrace{M_1 + M_2 + M_3}_{\text{Cut-set bound}}, \underbrace{2M_2 + 2M_3, 2N}_{\text{New bounds}}\}.$$

Transmission Strategy

Signal-space alignment for network-coding





- The signals $\mathbf{H_1x_1}$, $\mathbf{H_2x_2}$, and $\mathbf{H_3x_3}$ fill the entire space at the relay

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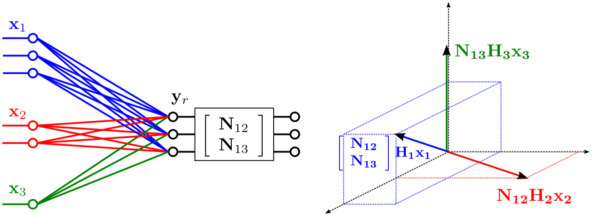

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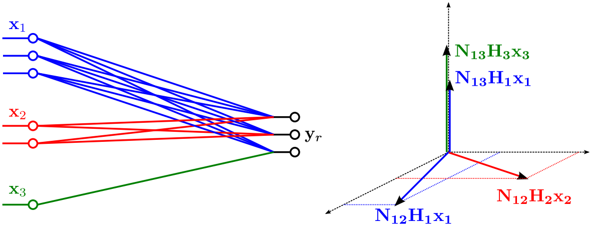

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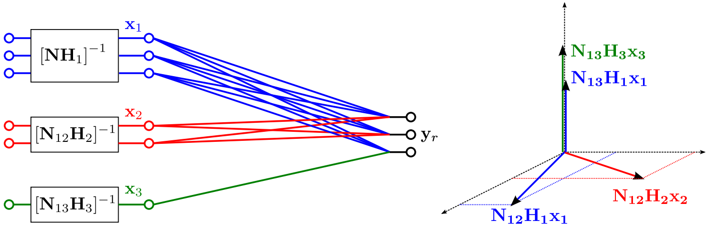

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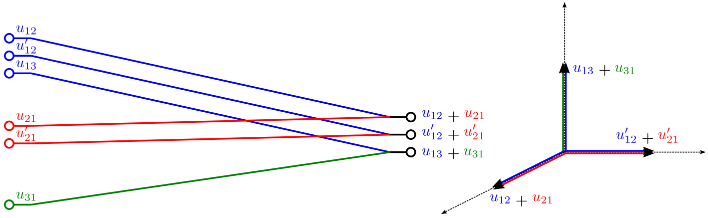

- The signals $\mathbf{H_1x_1}$, $\mathbf{H_2x_2}$, and $\mathbf{H_3x_3}$ fill the entire space at the relay
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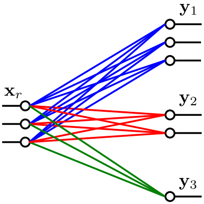
- Tx's diagonalize their effective channels $\Rightarrow$ desired channel structure
- Relay obtains net-coded signals: $u_{12}+u_{21}$, $u'_{12}+u'_{21}$, and $u_{13}+u_{31}$

Transmission strategy



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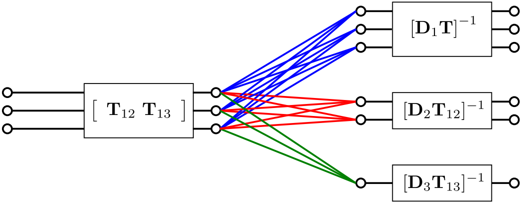

- Relay uses a similar beam-forming strategy to deliver the net-coded signals to their desired destinations

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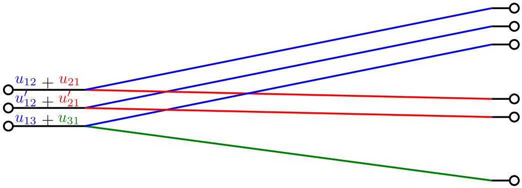

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- User 2 gets $u_{12} + u_{21}$ and $u'_{12} + u'_{21}$ and extracts u_{12} and u'_{12}
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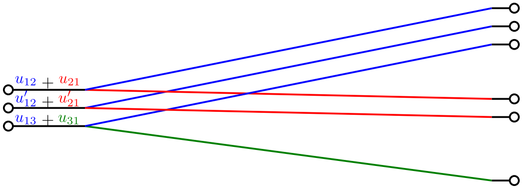
Transmission strategy



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- 6 symbols delivered successfully $\Rightarrow$ 6 DoF (optimal)

General Transmission Strategy

Case 1: $d = \min\{2M_2 + 2M_3, M_1 + M_2 + M_3, 2N\} = 2M_2 + 2M_3$

- Use only $M_2 + M_3$ antennas at the relay

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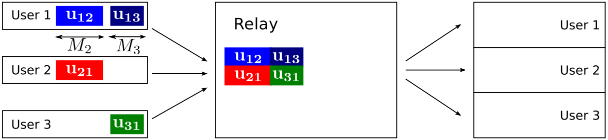

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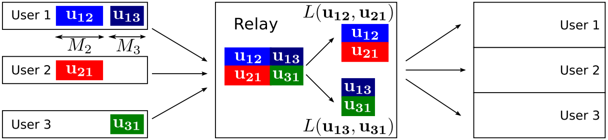

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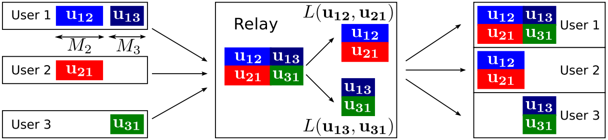

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- Beam-form $L(\mathbf{u}_{12}, \mathbf{u}_{21})$ and $L(\mathbf{u}_{13}, \mathbf{u}_{31})$ orthogonal to users 3 and 2, respectively

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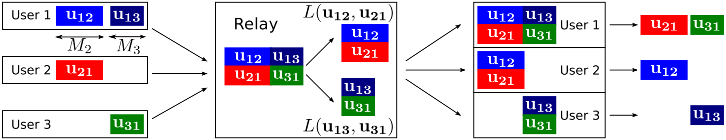

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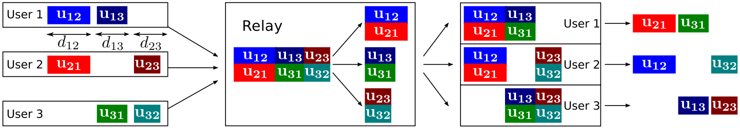
- Beam-form $L(\mathbf{u}_{12}, \mathbf{u}_{21})$ and $L(\mathbf{u}_{13}, \mathbf{u}_{31})$ orthogonal to users 3 and 2, respectively
- Each user decodes the desired linear combinations, and extracts the desired signals $\Rightarrow 2M_2 + 2M_3$ DoF

General Transmission Strategy



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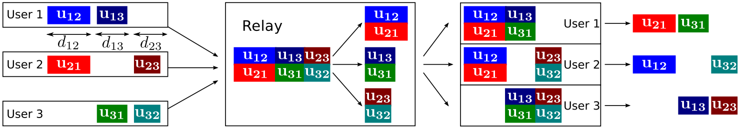
- Similar to [Lee *et al.* 10] but with an asymmetric DoF allocation

General Transmission Strategy



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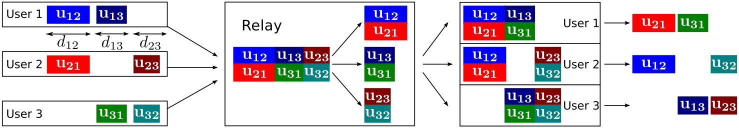

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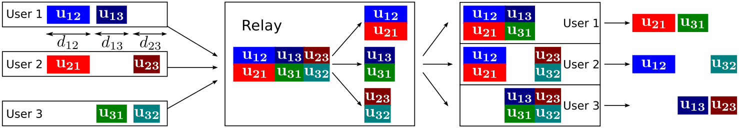
General Transmission Strategy



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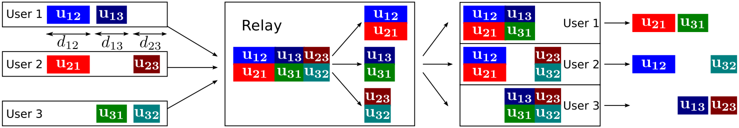
General Transmission Strategy



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- Achieve $d = 2d_{12} + 2d_{13} + 2d_{23} = M_1 + M_2 + M_3$ DoF

General Transmission Strategy

Case 3: $d = \min\{2M_2 + 2M_3, M_1 + M_2 + M_3, 2N\} = 2N$

- Reduce the number of antennas at the users so that $N = \min\{2M_2 + 2M_3, M_1 + M_2 + M_3\}$

General Transmission Strategy

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General Transmission Strategy

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Remark: If $M_3 = 0 \Rightarrow$ sum-DoF of the two-way relay channel

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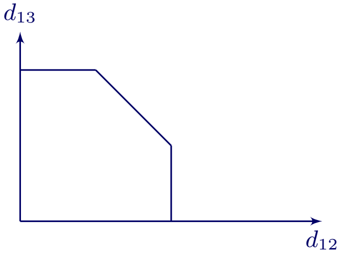

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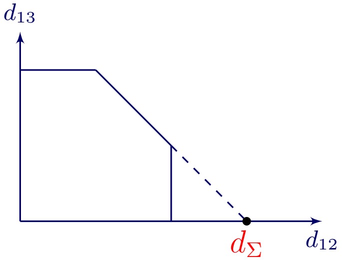

+ d_Σ is an overall measure of performance for a network

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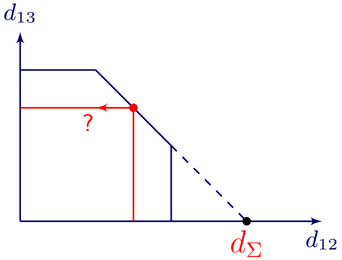
d_{ij}: DoF defined as $\lim_{P\to\infty} \frac{R_{ij}}{C(P)}$

d_{Σ}: sum-DoF $= \sum d_{ij}$

$\mathcal{D}$: Set of simultaneously achievable DoF's d_{ij}



+ d_{Σ} is an overall measure of performance for a network
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Importance of DoF region

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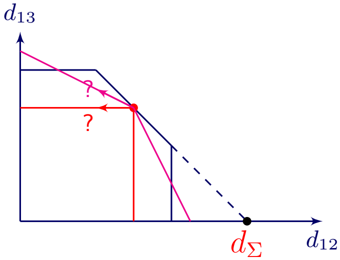

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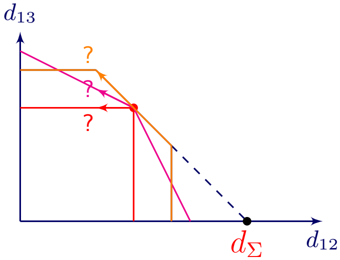

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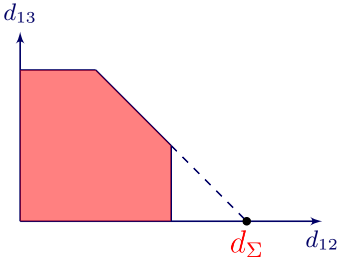

+ d_Σ is an overall measure of performance for a network
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Goal

Find the DoF region of the MIMO Y-channel.

DoF Region

- Bi-directional communication suffices for sum-DoF,

DoF Region

- Bi-directional communication suffices for sum-DoF,
- True for DoF-region?

DoF Region

- Bi-directional communication suffices for sum-DoF,
- True for DoF-region?
- No!

Optimal scheme is a combination of bi-directional, cyclic, and uni-directional schemes.

Simple example

Goal: Achieve the DoF tuple: over a Y-channel with $M = N = 3$

d_{12}	d_{13}	d_{21}	d_{23}	d_{31}	d_{32}
2	0	1	1	1	0

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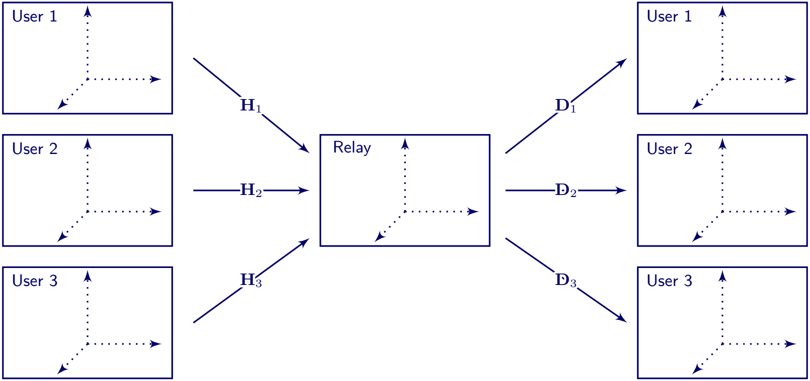
Lets try uni-directional



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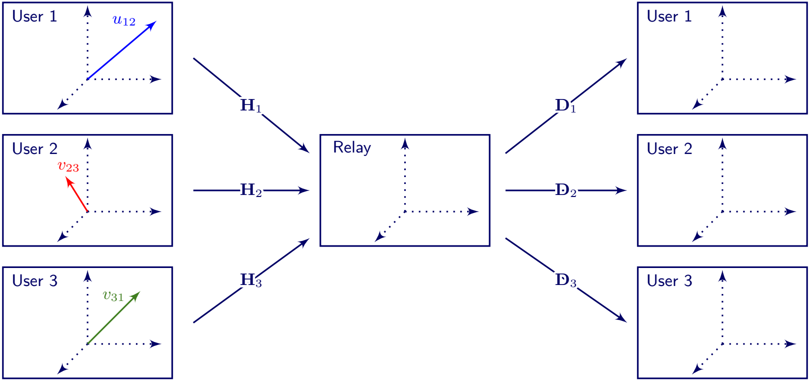

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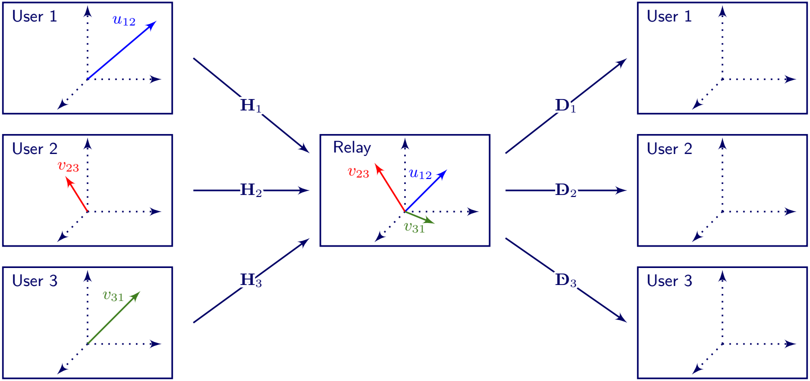

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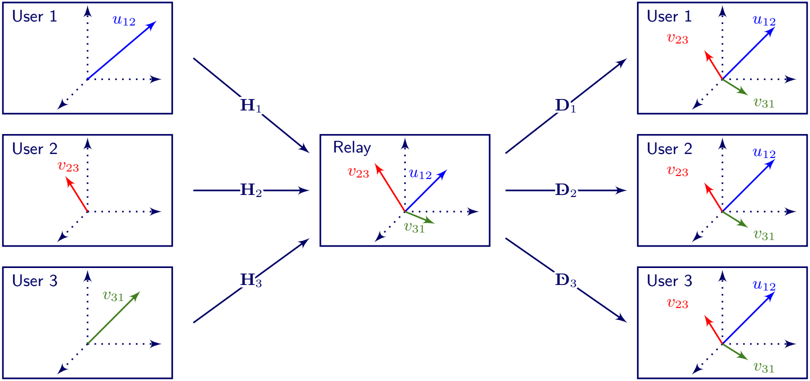

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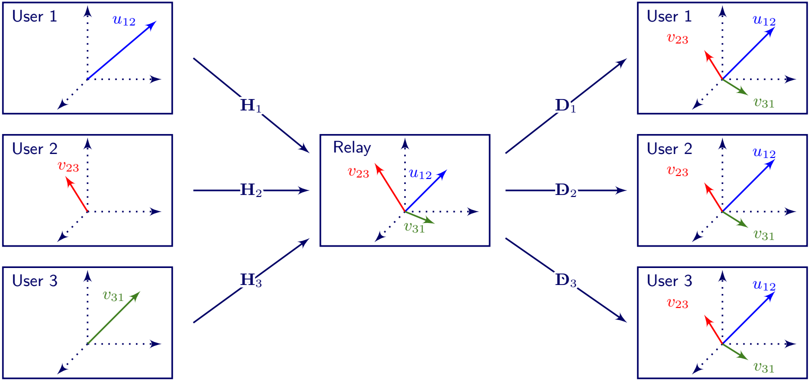

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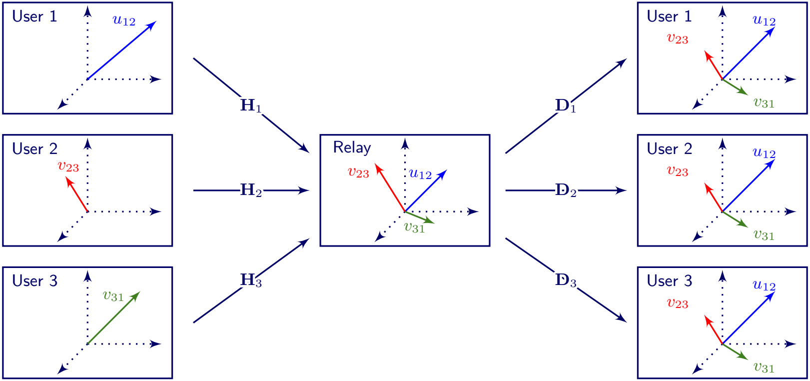
Lets try uni-directional+ bi-directional



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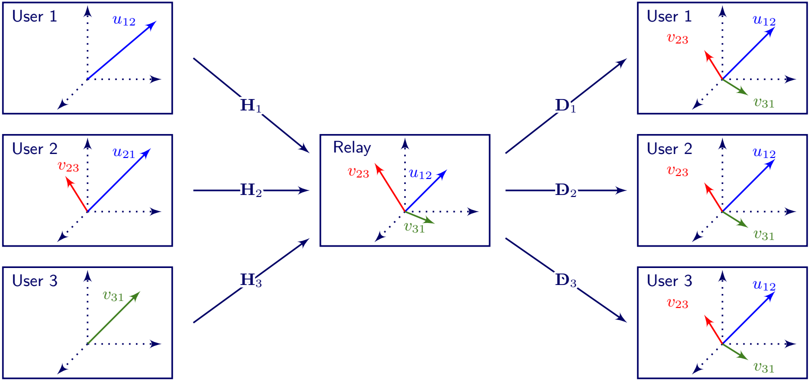

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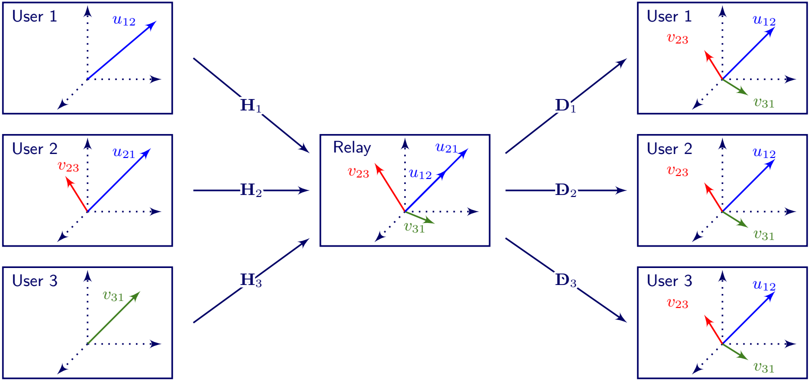

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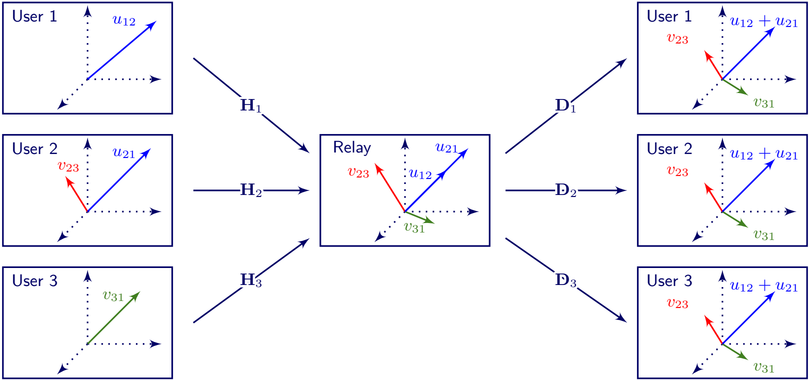

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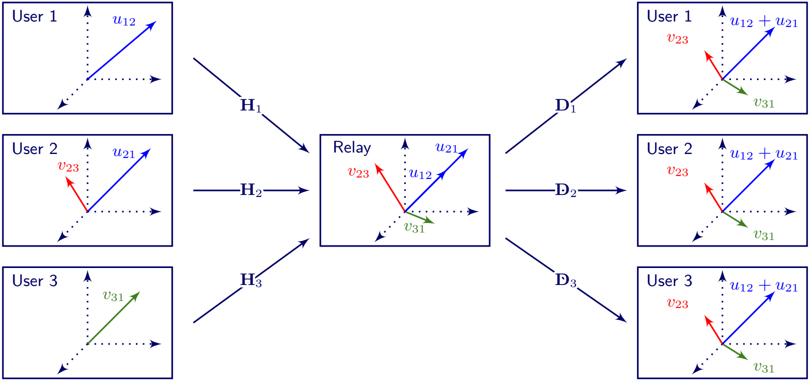

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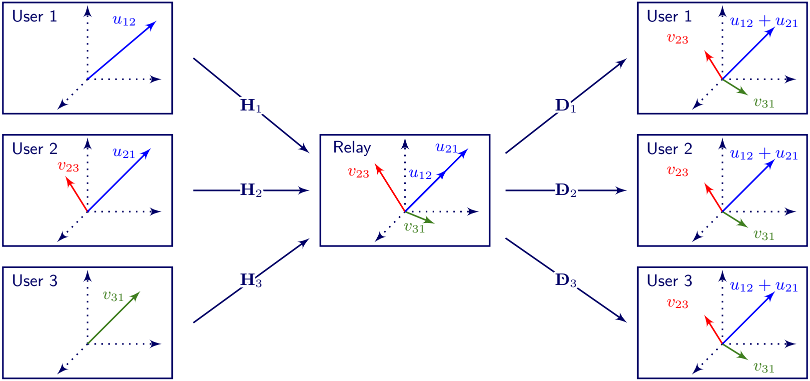
Lets try uni-directional+ bi-directional+ Cyclic



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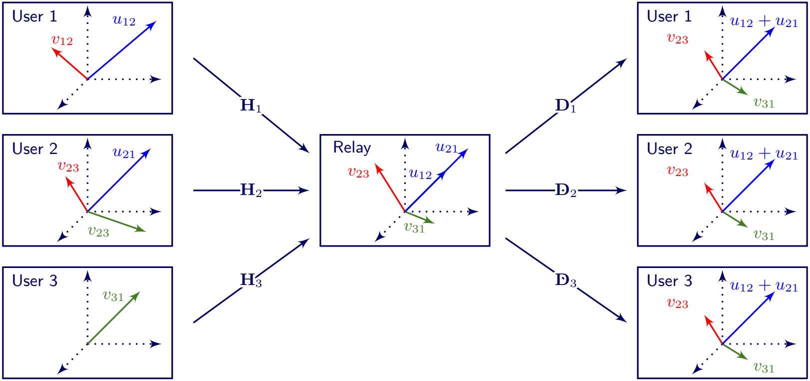

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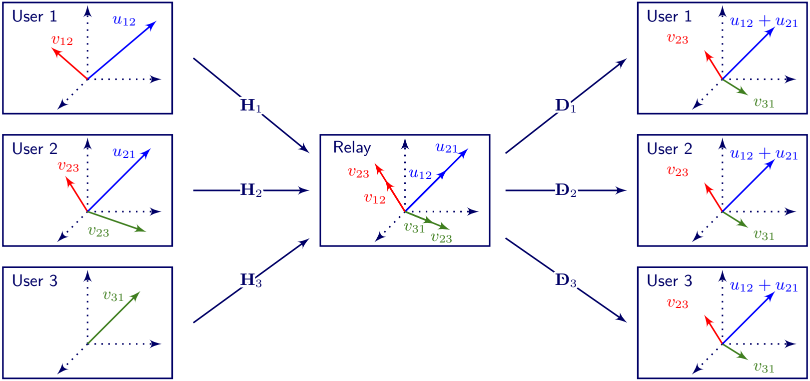

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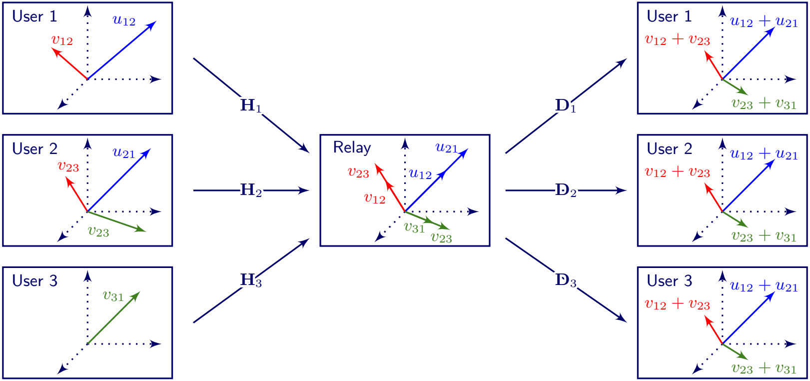

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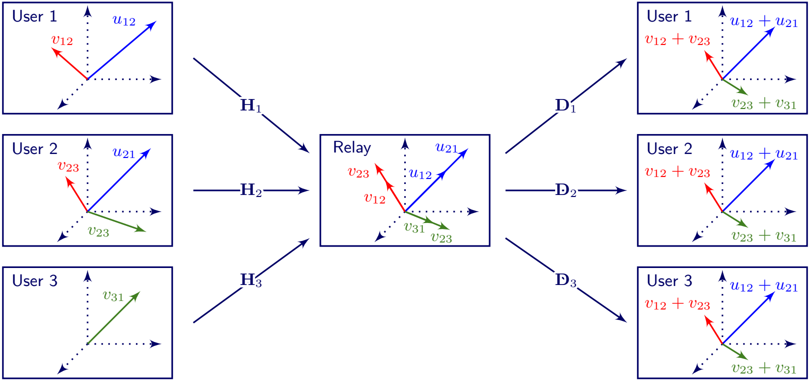

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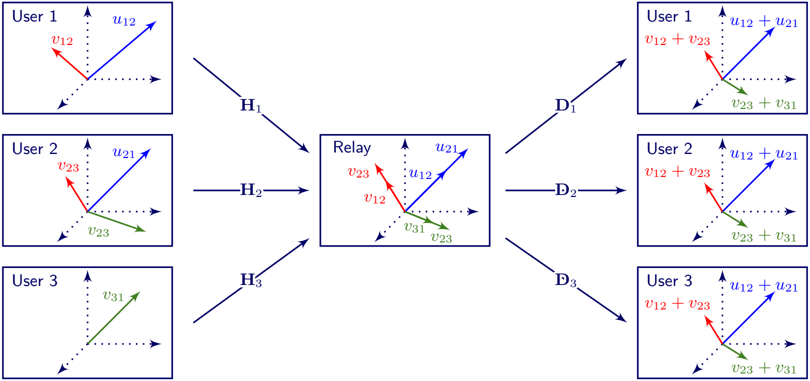
Lets try uni-directional + bi-directional + Cyclic $\leftarrow$ optimal combination



DoF Region

The DoF region of a 3-user MIMO Y-channel with $N \leq M$ is described by

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DoF region [C. & S. 2014]

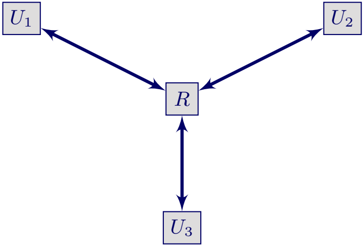
DoF region for $N \leq M$ described by

$$d_{p_1p_2} + d_{p_1p_3} + d_{p_2p_3} \leq N, \quad \forall \mathbf{p}$$

where $\mathbf{p}$ is a permutation of $(1, 2, 3)$ and p_i is its i-th component.

Overview

Achievability of $\mathcal{D}$ is proved using:



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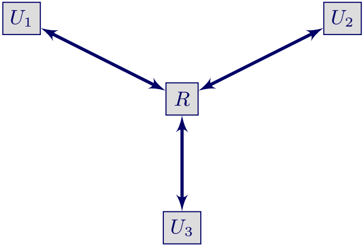
Channel diagonalization:

MIMO
Y-channel



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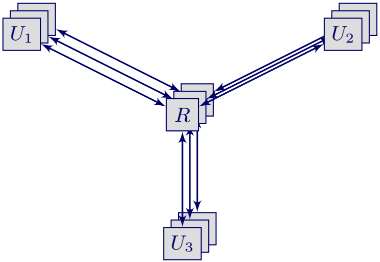
Channel diagonalization:

MIMO Y-channel $\rightarrow$ N SISO Y-channels (sub-channels)



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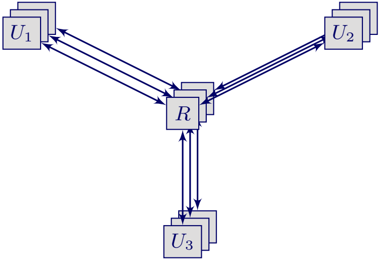

Information exchange:

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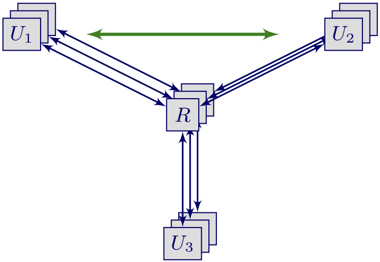

Information exchange:

- Bi-directional: signal-alignment/compute-forward

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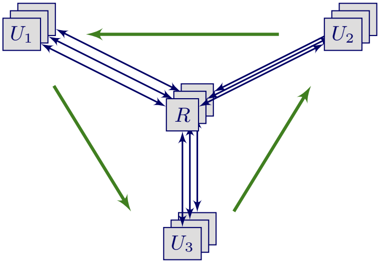

Channel diagonalization:

MIMO Y-channel $\rightarrow$ N SISO Y-channels (sub-channels)

Information exchange:

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- Cyclic: signal-alignment/compute-forward

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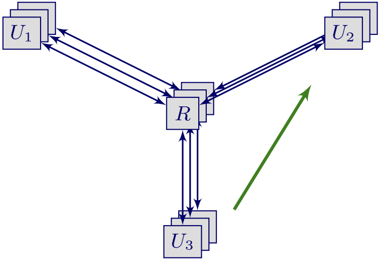

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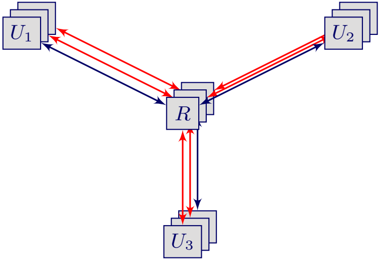

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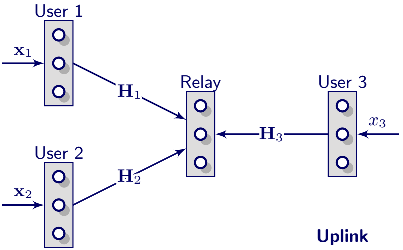
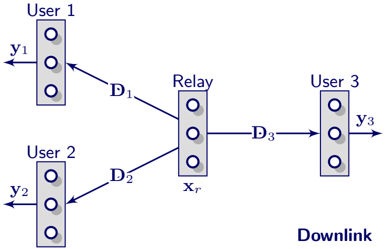
Resource allocation: distribute sub-channels over users

Channel Diagonalization

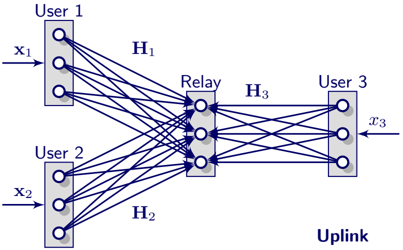
- a MIMO Y-channel with $M = N = 3$

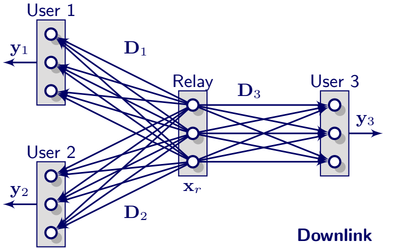




Channel Diagonalization

- a MIMO Y-channel with $M = N = 3$
- actually looks like this!





Channel Diagonalization

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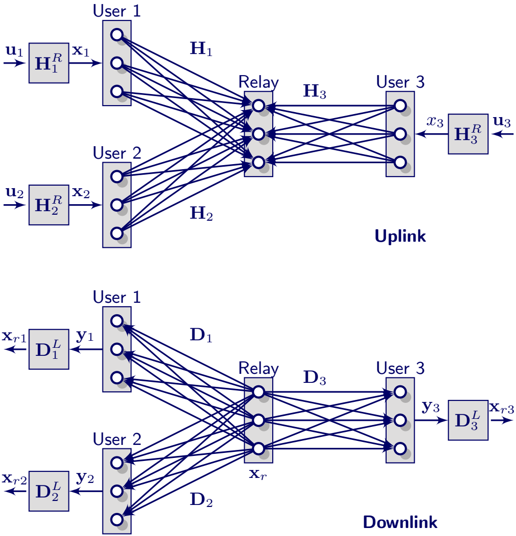
- Pre- and post-code using the Moore-Penrose pseudo inverse



Channel Diagonalization

- a MIMO Y-channel with $M = N = 3$
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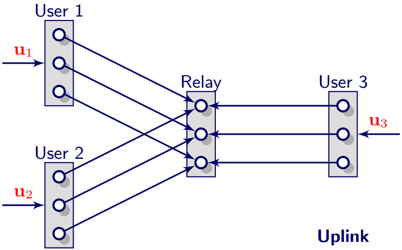
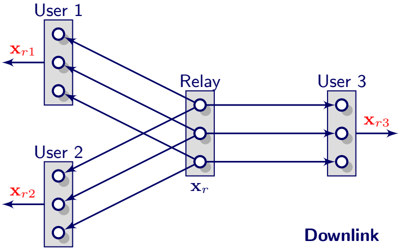
- Pre- and post-code using the Moore-Penrose pseudo inverse
- Channel Diagonalization $\Rightarrow N$ sub-channels

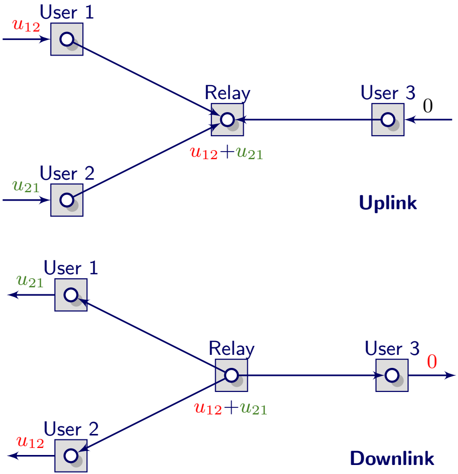




Information transfer

Bi-directional:

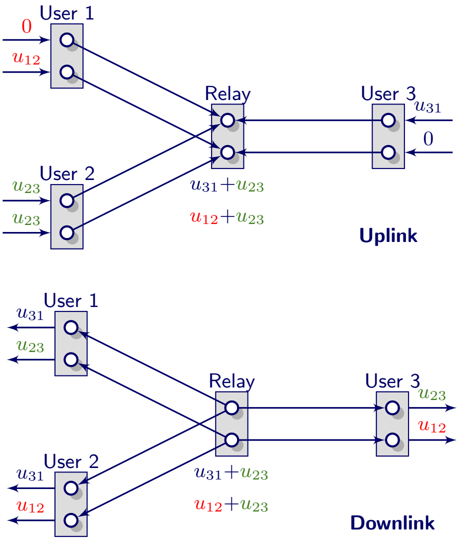
- signal-alignment
- compute-forward
- exchanges 2 symbols
- requires 1 sub-channel (up- and down-link)
- efficiency 2 DoF/dimension



Information transfer

Cyclic:

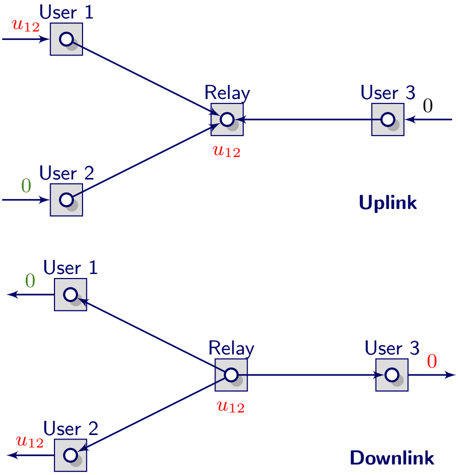
- signal-alignment
- compute-forward
- exchanges 3 symbols
- requires 2 sub-channels (up- and down-link)
- efficiency 3/2 DoF/dimension



Information transfer

Uni-directional:

- decode-forward
- exchanges 1 symbols
- requires 1 sub-channel (up- and down-link)
- efficiency 1 DoF/dimension



Back to our example

DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}) = (2, 0, 1, 1, 1, 0)$, Y-channel with $3 = N \leq M$

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Uni-directional only:

- 5 sub-channels $> N$!

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Bi-directional + uni-directional:

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- bi-directional achieves $d_{12}^b = d_{21}^b = 1$ over 1 sub-channel

Back to our example

DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}) = (2, 0, 1, 1, 1, 0)$, Y-channel with $3 = N \leq M$

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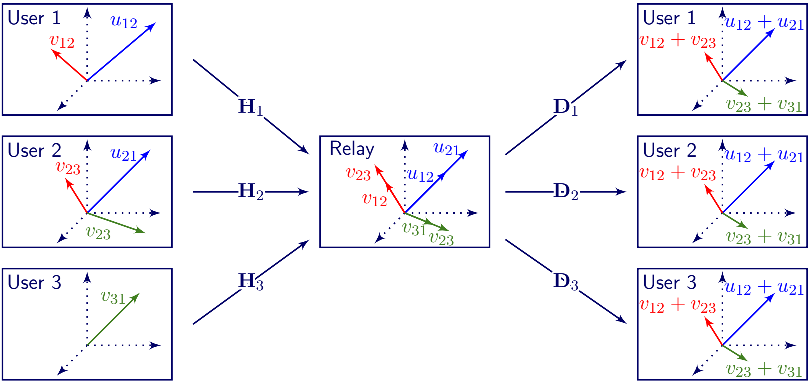

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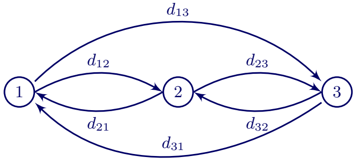

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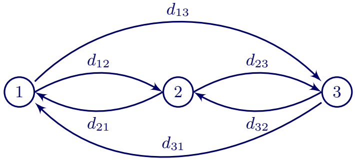

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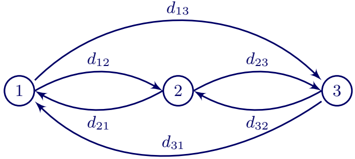

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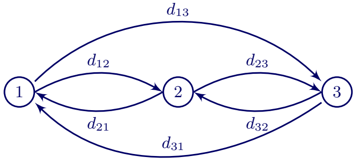

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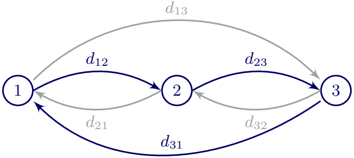

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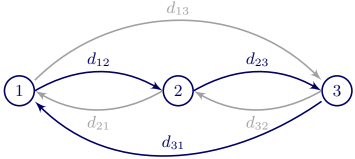

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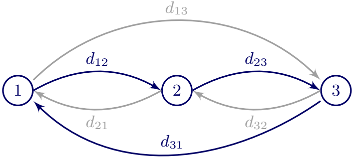

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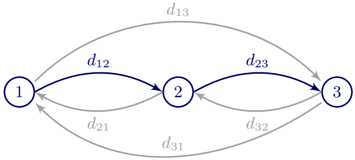

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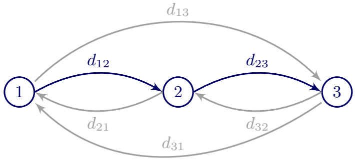

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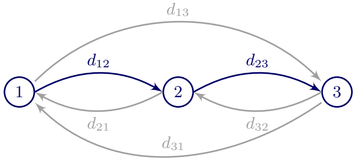

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$\mathbf{d}$ achieved!

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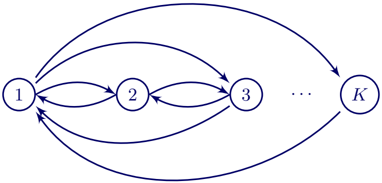

K-user Case

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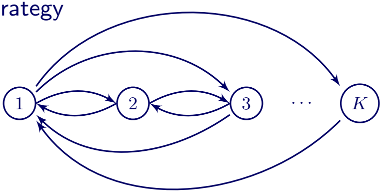

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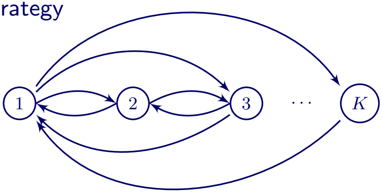

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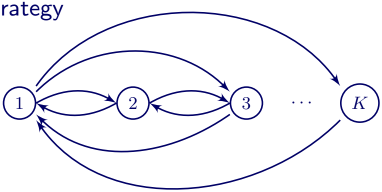

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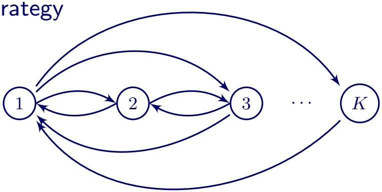

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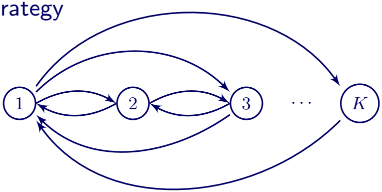

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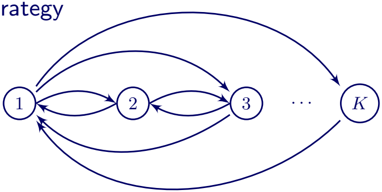

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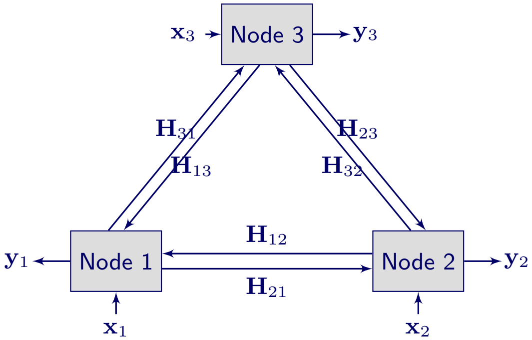
Cyclic communication requires joint encoding over multiple sub-channels⇒ MIMO Y-channels are in general inseparable!

Outline

1. From Capacity to DoF
2. MIMO Two-Way Relay Channel
   - Channel diagonalization
   - Signal Alignment
3. MIMO multi-way relay channel
   - Sum-DoF
   - DoF Region
4. MIMO Multi-way Channel

MIMO 3-Way Channel



- $\mathbf{y}_k = \mathbf{H}_{kj}\mathbf{x}_j + \mathbf{H}_{ki}\mathbf{x}_i + \mathbf{z}_k$,

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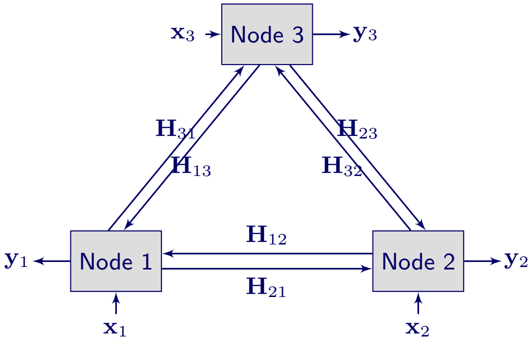

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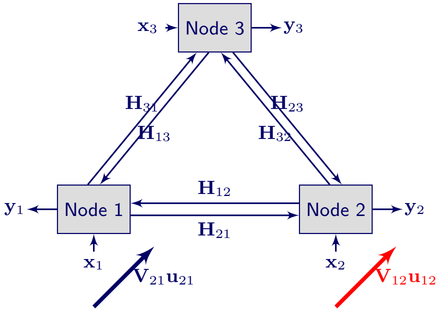
- Capacity scaling (DoF)?

Transmission Scheme



Communication $i \leftrightarrow j$:

- Node i sends $\mathbf{x}_i = \mathbf{V}_{ji}\mathbf{u}_i$, node j sends $\mathbf{x}_j = \mathbf{V}_{ij}\mathbf{u}_j$

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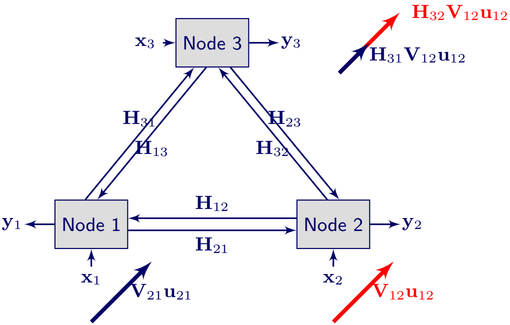

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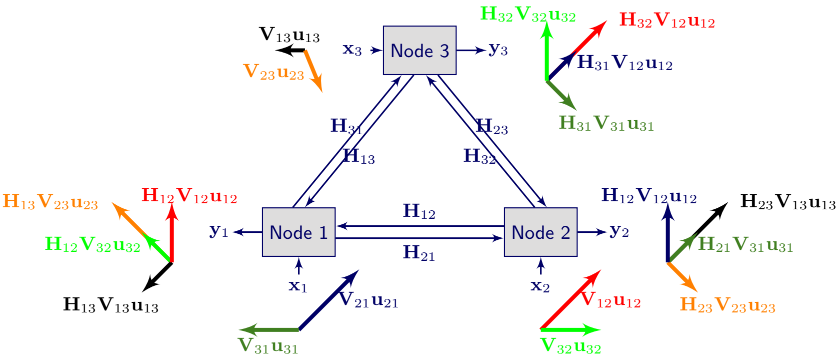

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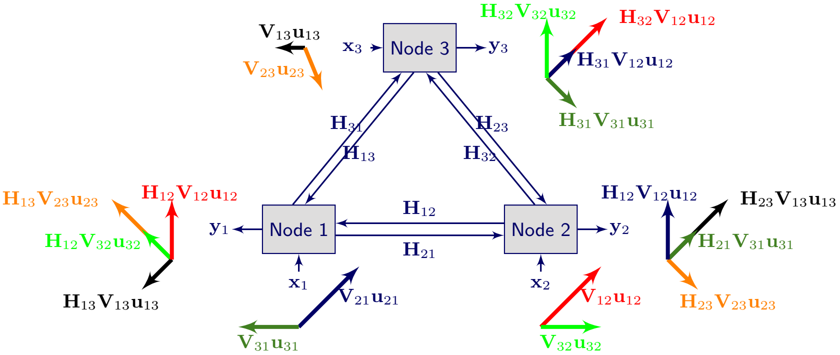

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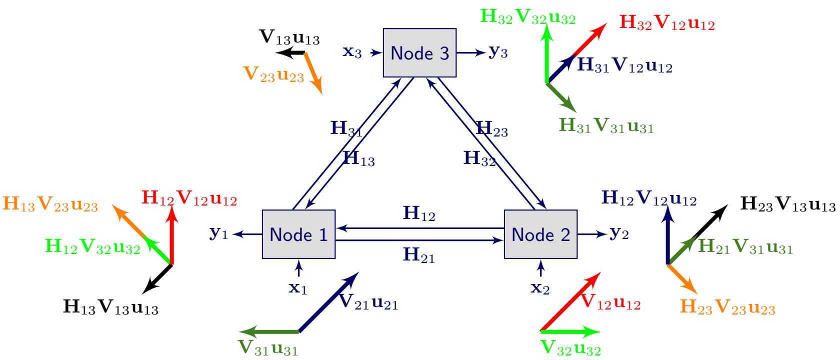

Achievable DoF:

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- Optimal scheme (genie-aided bound)

Summary

Two-way Relay Channel:

- Quantize-forward achieves similar rate as CF [Avestimehr *et al.* 10]
- Capacity region known within a constant gap [Nam *et al.* 10]
- Capacity of the BC phase is known [Oechtering *et al.* 08]
- Fading and scheduling [Shaqfeh *et al.* 13]
- Impact of CSIT [Yang *et al.* 13]
- Impact of direct channels [Avestimehr *et al.* 10]
- Energy harvesting [Tutuncuoglu *et al.* 13]
- Multiple relays [Vaze & Heath 09]

Multi-way Relay Channel:

- 3-user LD case with relay private messages, and 4-user LD case [Zewail *et al.* 13]
- Direct links between users [Lee & Heath 13]
- Multi-cast setting: compress-forward [Gündüz *et al.* 13], compute-forward [Ong *et al.* 12]
- Fading case [Wang *et al.* 12]

Multi-way Channel:

- Capacity of classes of 3-way channels [Ong 12]
- Two-way interference channel [Rost 11]
- Two-way IC (feedback better than info. transmission!) [Suh *et al.* 13]
- Two-way networks (MAC,BC,TWC) [Cheng & Devroye 14]

MIMO Two-way Relay Channel:

- Diversity-multiplexing trade-off [Gündüz *et al.* 08], [Vaze & Heath 11],
- Cognition, multiple relays [Alsharoa *et al.* 13],
- Multi-pair sum-rate optimization in [S. *et al.* 09],
- DoF of the K-pair case [Lee & Heath 13], [Cheng & Devroye 13],
- Imperfect CSI [Ubaidulla *et al.* 13], [Zhang *et al.* 13],

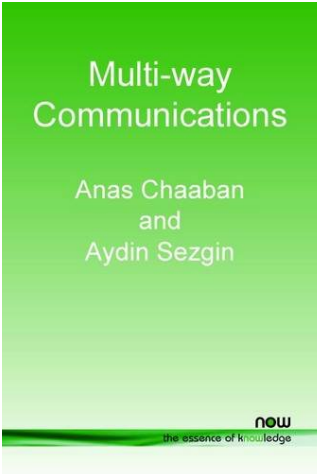
MIMO Multi-way Relay Channel:

- Multi-cluster multi-way relay channels [Tian & Yener 12],
- Performance optimization [Teav *et al.* 14],
- K-user achievable sum-DoF [Lee *et al.* 12],
- Full- vs. half-duplex, global vs. local CSI [Lee & Chun 11],

Interesting Problems

- Optimizing the uplink in the two-way relay channel
- Exploiting two-way relaying in larger networks
- Constellations for PLNC and their performance
- Multi-relay cases
- General (approximate) capacity expression for the K-user multi-way relay channel
- Extensions of the multi-way channel to K-users
- Fading multi-way channels
- Capacity region study of the MIMO two-way relay channel
- Sum-DoF of K-user multi-way relay channels (4-user case characterized recently [Wang 14])
- Rate maximization/power minimization,
- Self-interference cancellation techniques and their impact

All this and more to appear in:



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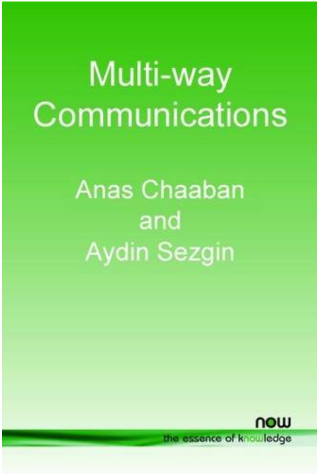

Thank you for your attention!

Optimality

Bi-directional	2 symbols	1 sub-channel
Cyclic	3 symbols	2 sub-channels
Uni-directional	1 symbol	1 sub-channel

Optimality

Total number of dimensions required to achieve $\mathbf{d} \in \mathcal{D}$:

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No cycles $\Rightarrow N_s \leq N$

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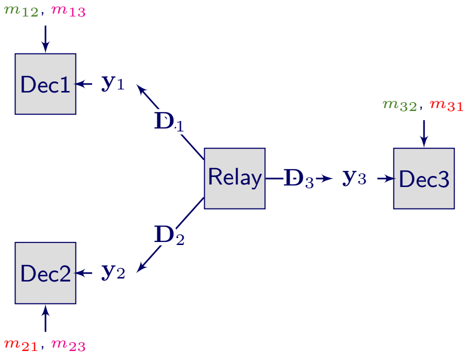
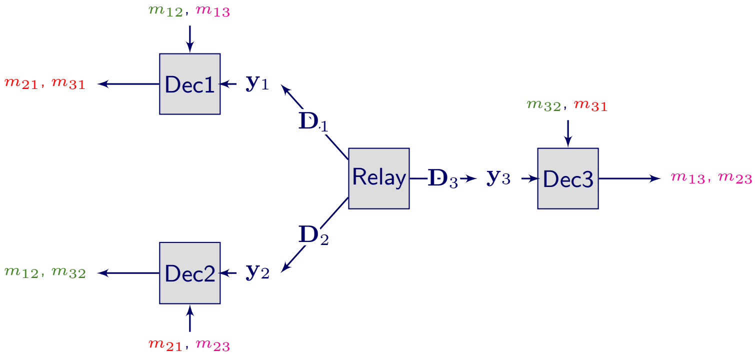
No cycles $\Rightarrow N_s \leq N \Rightarrow$ All $\mathbf{d} \in \mathcal{D}$ are achievable

Outer bound



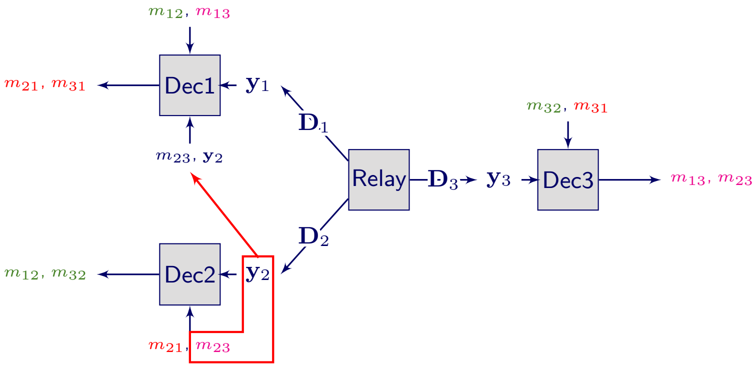
Consider any reliable scheme for the 4-user MIMO MRC

Outer bound



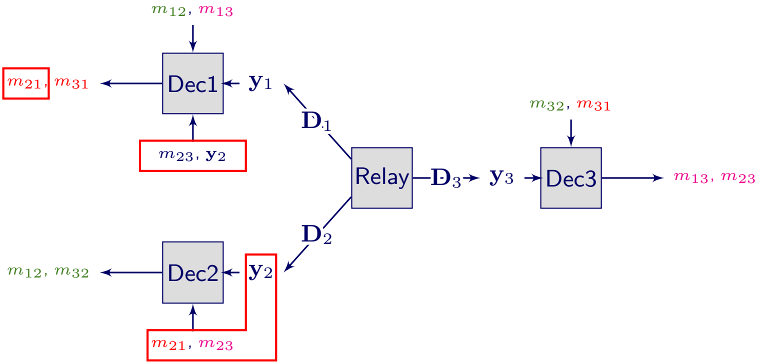
Users can decode their desired signals

Outer bound



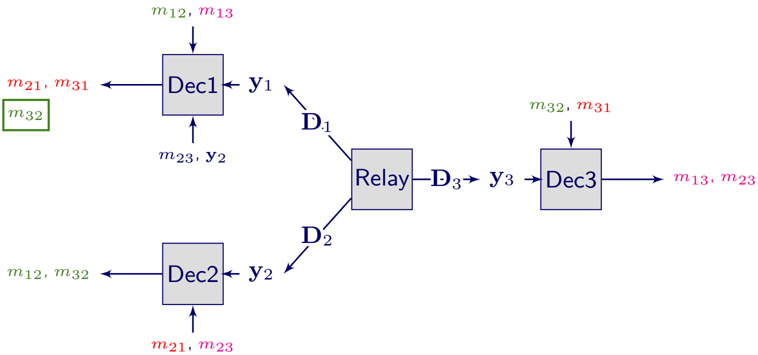
Give m_{23} and $\mathbf{y}_2$ to user 1 as side info.

Outer bound



Now, user 1 has the info. available at user 2

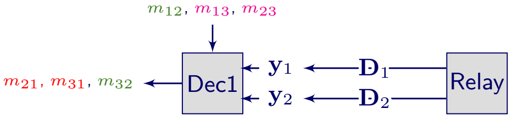
Outer bound



$\Rightarrow$ User 1 can decode m_{32}

Upper bound

User 1 can decode (m_{21}, m_{31}, m_{32}) from $(m_{12}, m_{13}, \mathbf{y}_1, \overbrace{m_{23}, \mathbf{y}_2}^{\text{side info.}})$



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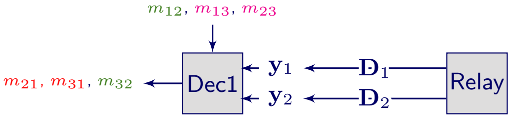

$$\Rightarrow R_{21} + R_{31} + R_{32} \leq I(\mathbf{x}_r; \mathbf{y}_1, \mathbf{y}_2) = I\left(\mathbf{x}_r; \begin{bmatrix}\mathbf{D}_1\\ \mathbf{D}_2\end{bmatrix}\mathbf{x}_r + \begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{bmatrix}\right)$$ P2P Channel

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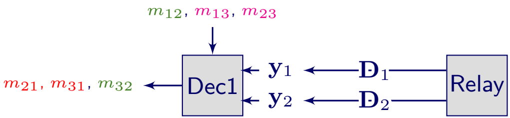

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P2P Channel

$$\Rightarrow d_{21} + d_{31} + d_{32} \leq \text{rank}\left(\begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \end{bmatrix}\right) = N$$

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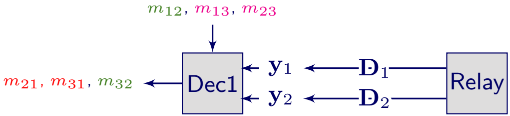

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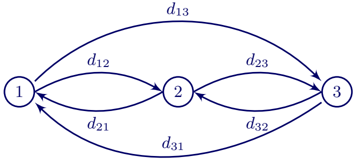
$$\Rightarrow d_{21} + d_{31} + d_{32} \leq \text{rank}\left(\begin{bmatrix}\mathbf{D}_1 \\ \mathbf{D}_2\end{bmatrix}\right) = N$$

Considering different combinations of users gives the desired outer bound

$$\sum_{i=1}^{2} \sum_{j=i+1}^{3} d_{p_i p_j} \leq N, \quad \forall \mathbf{p}$$

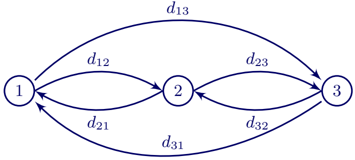
Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32})$



Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}) \Rightarrow$ 2-cycles and 3-cycles!



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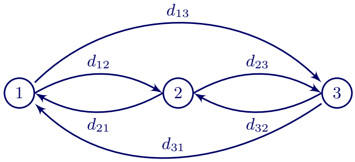
Bi-directional:

1) set $d^b_{ij} = d^b_{ji} = \min\{d_{ij}, d_{ji}\}$



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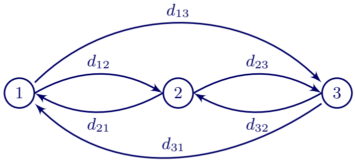
1) set $d^b_{ij} = d^b_{ji} = \min\{d_{ij}, d_{ji}\}$
2) requires d^b_{ij} sub-channels



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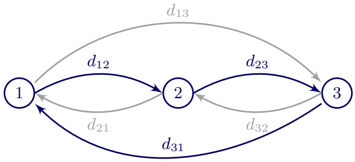
1) set $d_{ij}^b = d_{ji}^b = \min\{d_{ij}, d_{ji}\}$
2) requires d_{ij}^b sub-channels
3) resolves 2-cycles

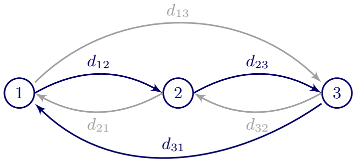


Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32})$ $\Rightarrow$ 2-cycles and 3-cycles!

Bi-directional:

1) set $d^b_{ij} = d^b_{ji} = \min\{d_{ij}, d_{ji}\}$
2) requires d^b_{ij} sub-channels
3) resolves 2-cycles
4) residual DoF $d'_{ij} = d_{ij} - d^b_{ij}$



Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32})$ ⇒ 2-cycles and 3-cycles!

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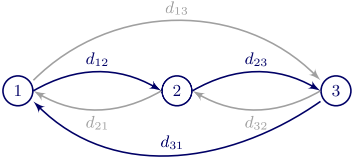

Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0)$ ⇒ 3-cycle!

Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}) \Rightarrow$ 2-cycles and 3-cycles!

Bi-directional:

1) set $d^b_{ij} = d^b_{ji} = \min\{d_{ij}, d_{ji}\}$
2) requires d^b_{ij} sub-channels
3) resolves 2-cycles
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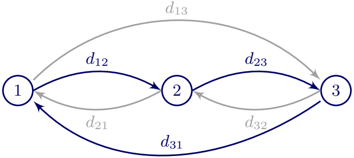


Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0) \Rightarrow$ 3-cycle!

Cyclic:

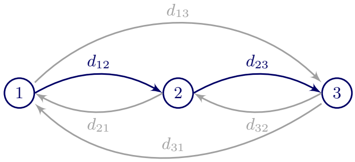
1) set $d^c_{ij} = d^c_{jk} = d^c_{ki} = \min\{d'_{ij}, d'_{jk}, d'_{ki}\}$

Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32})$ ⇒ 2-cycles and 3-cycles!

Bi-directional:

1) set $d^b_{ij} = d^b_{ji} = \min\{d_{ij}, d_{ji}\}$
2) requires d^b_{ij} sub-channels
3) resolves 2-cycles
4) residual DoF $d'_{ij} = d_{ij} - d^b_{ij}$



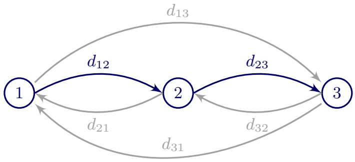
Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0)$ ⇒ 3-cycle!

Cyclic:

1) set $d^c_{ij} = d^c_{jk} = d^c_{ki} = \min\{d'_{ij}, d'_{jk}, d'_{ki}\}$
2) requires $2d^c_{ij}$ sub-channels

Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}) \Rightarrow$ 2-cycles and 3-cycles!



Bi-directional:

1) set $d^b_{ij} = d^b_{ji} = \min\{d_{ij}, d_{ji}\}$
2) requires d^b_{ij} sub-channels
3) resolves 2-cycles
4) residual DoF $d'_{ij} = d_{ij} - d^b_{ij}$

Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0) \Rightarrow$ 3-cycle!

Cyclic:

1) set $d^c_{ij} = d^c_{jk} = d^c_{ki} = \min\{d'_{ij}, d'_{jk}, d'_{ki}\}$
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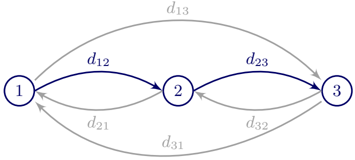
3) resolves 3-cycles

Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32})$ $\Rightarrow$ 2-cycles and 3-cycles!



Bi-directional:

1) set $d^b_{ij} = d^b_{ji} = \min\{d_{ij}, d_{ji}\}$
2) requires d^b_{ij} sub-channels
3) resolves 2-cycles
4) residual DoF $d'_{ij} = d_{ij} - d^b_{ij}$

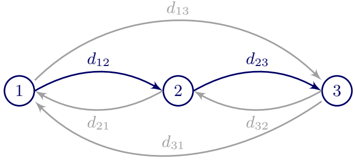
Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0)$ $\Rightarrow$ 3-cycle!

Cyclic:

1) set $d^c_{ij} = d^c_{jk} = d^c_{ki} = \min\{d'_{ij}, d'_{jk}, d'_{ki}\}$
2) requires $2d^c_{ij}$ sub-channels
3) resolves 3-cycles
4) residual DoF $d''_{ij} = d'_{ij} - d^c_{ij}$

Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32})$ ⇒ 2-cycles and 3-cycles!



Bi-directional:

1) set $d_{ij}^b = d_{ji}^b = \min\{d_{ij}, d_{ji}\}$
2) requires d_{ij}^b sub-channels
3) resolves 2-cycles
4) residual DoF $d'_{ij} = d_{ij} - d_{ij}^b$

Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0)$ ⇒ 3-cycle!

Cyclic:

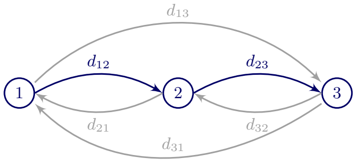
1) set $d_{ij}^c = d_{jk}^c = d_{ki}^c = \min\{d'_{ij}, d'_{jk}, d'_{ki}\}$
2) requires $2d_{ij}^c$ sub-channels
3) resolves 3-cycles
4) residual DoF $d''_{ij} = d'_{ij} - d_{ij}^c$

Uni-directional:

1) set $d_{ij}^u = d''_{ij}$

Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32})$ ⇒ 2-cycles and 3-cycles!



Bi-directional:

1) set $d^b_{ij} = d^b_{ji} = \min\{d_{ij}, d_{ji}\}$
2) requires d^b_{ij} sub-channels
3) resolves 2-cycles
4) residual DoF $d'_{ij} = d_{ij} - d^b_{ij}$

Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0)$ ⇒ 3-cycle!

Cyclic:

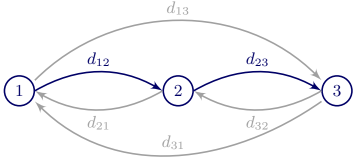
1) set $d^c_{ij} = d^c_{jk} = d^c_{ki} = \min\{d'_{ij}, d'_{jk}, d'_{ki}\}$
2) requires $2d^c_{ij}$ sub-channels
3) resolves 3-cycles
4) residual DoF $d''_{ij} = d'_{ij} - d^c_{ij}$

Uni-directional:

1) set $d^u_{ij} = d''_{ij}$
2) requires d^u_{ij} sub-channels

Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}) \Rightarrow$ 2-cycles and 3-cycles!



Bi-directional:

1) set $d_{ij}^b = d_{ji}^b = \min\{d_{ij}, d_{ji}\}$
2) requires d_{ij}^b sub-channels
3) resolves 2-cycles
4) residual DoF $d'_{ij} = d_{ij} - d_{ij}^b$

Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0) \Rightarrow$ 3-cycle!

Cyclic:

1) set $d_{ij}^c = d_{jk}^c = d_{ki}^c = \min\{d'_{ij}, d'_{jk}, d'_{ki}\}$
2) requires $2d_{ij}^c$ sub-channels
3) resolves 3-cycles
4) residual DoF $d''_{ij} = d'_{ij} - d_{ij}^c$

Uni-directional:

1) set $d_{ij}^u = d''_{ij}$
2) requires d_{ij}^u sub-channels

$\mathbf{d}$ achieved!