A Consistent Modeling of Passive Memcapacitive Systems

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Abstract—Memelements - circuit elements with memory - serve novel applications in several technical disciplines. Due to similar functionalities to synapses, they are especially suitable for neuromorphic circuits. Physical and chemical phenomena in the nanoscale lead to the unique information storage characteristic of these elements. Fabrication of devices with memory considering a particular desired functionality is still difficult to achieve. Therefore, simulation models based on a consistent modeling approach are needed. A consistent model in this context should consider important energetic properties of the real device, e.g. passivity.

We propose a novel circuit theoretic approach for a consistent modeling of lossless memcapacitive devices. They can be interpreted as nonlinear capacitances with memory. A comparison with existing modeling approaches of such elements underlines benefits as well as the necessity of a consistent model. The strategy introduced here is more general and independent of the underlying model. Beside simulations, it can also be utilized in emulations regarding real-time capable implementations.

I. INTRODUCTION

Memcapacitive devices are nonlinear capacitances with memory. They are, apart from meminductive devices, the reactive counterparts to memristive systems which are essentially nonlinear resistors with memory [1]. Such devices gained significant attention of several researchers since the first memristive device was realized in Hewlett-Packard’s laboratory in 2008 [2], especially in the area of neuromorphic circuits [3]. Reactive elements with memory offer further possibilities with unique functionalities. In particular, memcapacitive systems possess a practical relevance especially in neuromorphic applications [4]. Although there exist devices in nanotechnology which behave like capacitances with a memory [5], fabrication of such devices is not common yet. Particularly, the development of real devices with a desired functionality is a challenging procedure. Exploiting mathematical models of memcapacitive systems in simulations can help for first insights [6], [7].

Modeling of memristive systems while preserving energetic properties of the real device, like passivity, is prevalent [6], [8]. In contrast to memristive systems, the situation changes completely, if we deal with memory devices being reactive, like memcapacitive devices. In this case, the passivity can only be evaluated for an energetic definition of such elements [9]. Consequently, we are looking for a consistent modeling approach of memcapacitive devices which considers the passivity property without any restrictions to model parameters and the excitation. In fact, memcapacitive devices represented here are also lossless. Therefore, the circuit theory as an appropriate modeling tool has been utilized in order to get an as simple as possible parametric representation of lossless memcapacitive devices. Simulation results are compared to those from the literature by pointing out the differences.

It should be emphasized that the presented approach is not limited to one particular device model. Instead, it can be used for arbitrary modeling approaches. It can also be helpful for emulations of such systems, if we think about neuromorphic circuits with a huge number of devices which make simulations very time-consuming [10]–[12].

For the purpose of clarity, the next section recapitulates some definitions of nonlinear capacitance theory. Afterwards, we introduce a consistent modeling approach of memcapacitive devices followed by comparisons with usual model definitions from the literature. Main results are summarized in the conclusion.

II. NONLINEAR CAPACITORS

The modeling of memcapacitive devices is closely related to the modeling of nonlinear capacitors. Therefore, some definitions from nonlinear capacitor theory are recapitulated here.

The time dependence of a voltage-controlled capacitance yields several possibilities to extend the equation of a constant capacitor, like static, differential, and energetic capacitors

\[ C_s(u) = \frac{q}{u} , \quad \text{with} \quad i_s = \frac{dC_s(u)u}{du} \frac{du}{dt} \quad (1a) \]

\[ C_d(u) = \frac{dq}{du} , \quad \text{with} \quad i_d = C_d(u) \frac{du}{dt} \quad , (1b) \]

\[ C_e(u) = \frac{2E}{u^2} , \quad \text{with} \quad i_e = \sqrt{C_e(u)} \frac{d\sqrt{C_e(u)}u}{dt} \quad , (1c) \]

respectively, cf. [9], [13]. Here, \( q = q(u) \) is the voltage-controlled charge, whereas \( u \) denotes the voltage drop over the capacitor. Assuming passivity, the stored energy \( E \) should always be positive because the amount of removed energy...
cannot exceed the amount of previously added energy in passive systems
\[ E(t) = \int_{t_0}^{t} u(\tau) i(\tau) d\tau \geq 0, \quad \text{with} \quad E(t_0) = 0 \quad (2) \]
and \( t \geq t_0 \). These definitions for the voltage-controlled case are illustrated in Fig. 1.

It is shown that a positive device parameter for a static or a differential capacitance \( C_{a,d} \geq 0 \) cannot ensure the passivity, whereas, for an energetic definition, \( C_{e} \geq 0 \) is a necessary as well as sufficient condition for passivity [9]. Consequently, an energetic definition of a capacitance with respect to modeling memcapacitive systems is a more appropriate approach in order to get general statements about passivity.

III. Memcapacitive Systems

In this section, the insights from the nonlinear capacitor theory above have been utilized for a consistent modeling of memcapacitive systems. As mentioned in the introduction, such devices have a memory and hence they can be described by an internal state \( z \). For the sake of convenience, we have used a simple linear, flux-controlled memcapacitive model
\[ C_{\text{lin}}(\varphi) = C_0 + \Delta C [a + k \varphi], \]
with \( \Delta C = C_1 - C_0 \) and \( a = \frac{C_{\text{lin}}(0) - C_0}{C_1 - C_0} \),
where the internal state \( z \) is the flux \( \varphi \)
\[ z(t) = \varphi(t) = \varphi_0 + \int_{t_0}^{t} u(\tau) d\tau . \quad (4) \]

Here, \( k = 100 \) 1/Wb, \( C_1 = 100 \) pF and \( C_0 = 1 \) pF are model parameters. Indeed, in this simplified model the window function has been neglected and hence there are no limiting values. In doing so, the disadvantages and benefits of different modeling approaches can be extracted more obviously. For the initial state \( \varphi(t_0) = 0 = \varphi_0 = 0 \) is assumed in the following. The three definitions of memcapacitive devices are discussed in the sequel on this model.

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A. Lossless Memcapacitive System

For a consistent modeling of circuit elements with memory (memelements), energetic properties, like the passivity of the real device, should be maintained. While checking the passivity of memristive devices is unproblematic, it becomes challenging for reactive memelements, e.g. memcapacitive systems [14]. However, the passivity is ensured for \( C_{\text{lin}}(\varphi) = C_0(\varphi) > 0 \), considering an energetic definition (1c). A circuit theoretic interpretation of the current through an energetic memcapacitive device yields the electrical representation shown in Fig. 2, with \( C_0(\varphi) = C_0 \varphi^2(\varphi) \) and the normalization capacitance \( C = 1 \) F. Since the energy-neutral transformer and the lossless capacitor are port-wise interconnected via a Kirchhoff interconnection network, the resulting system is also lossless for \( C > 0 \). With this, the instantaneous power flow through the device and the stored energy can be summarized as
\[ p_e = i_e u = \frac{dE}{dt}, \quad \text{with} \quad E = \frac{1}{2} C_0 u^2 = \frac{1}{2} C v^2 \geq 0 . \quad (5) \]

Simulation results of this energy for an input voltage
\[ u(t) = u_0 \sin (2\pi f_0 t) + U_{\text{off}}, \quad (6) \]
with amplitude \( u_0 = 1 \) V and frequency \( f_0 = 10 \) Hz are depicted in Fig. 3. For comparison purposes with the static memcapacitive device which is discussed in the next section, two cases are investigated here, namely the input voltage with and without a constant offset \( U_{\text{off}} \). As expected from equation (5), the energy stored in the device is always positive for \( C > 0 \) and for an initial value of \( E(t_0) = 0 \).

One fingerprint of memelements is the pinched hysteresis loop caused by the memory. From equation (1c), it is evident...
that the hysteresis loop of an energetic memcapacitive system is evaluated in the \((E - u)\)-plane. This hysteresis loop is shown in Fig. 4 for \(U_{\text{off}} = 0\). In contrast to common hysteresis loops regarding circuit elements with memory, this curve has an axisymmetrical shape because of the quadratic form with respect to the applied voltage \(u\).

**B. Static Memcapacitive System**

The most popular memcapacitive devices in the literature are defined in a static matter. In accordance with equation (1a), the current of a static memcapacitive device

\[
i_s = i_e + G(\varphi) u, \quad \text{with} \quad G(\varphi) = \frac{1}{2} \frac{dC_{\text{lin}}(\varphi)}{dt} = \frac{1}{2} \Delta C k u \varphi
\]

(7)
is a superposition of the energetic current \(i_e\) with an amount depending on the time derivative of the memcapacitance. It can be interpreted as a conductance, cf. Fig 5. We notice that this conductance can be negative and hence the modeled device could fail to satisfy the passivity condition. The transient behavior of the conductance for the input voltage (6) without an offset is depicted in Fig. 6. A piecewise negative conductance is observed for the input voltage (6), with \(U_{\text{off}} = 0\). This result confirms previous thoughts about passivity. The static definition of a memcapacitive system is not passive without any restrictions to the input voltage as well as to model parameters. However, the average energy stored in the device is positive (Fig. 7, gray-dashed) with respect to a harmonic excitation, whereas an offset of \(U_{\text{off}} = -50\) mV leads to a negative stored energy (Fig. 7 - colored) for \(C > 0\) and hence passivity is no longer valid (not even externally), cf. Fig. 7. Unlike the static memcapacitive device, the energetic definition preserves the passivity for an arbitrary excitation with an offset, cf. Fig. 3.

As known from common memcapacitive models [7] and in accordance with equation (1a), the \((q - u)\)-plane yields a representation for the static memcapacitive hysteresis loop depicted in Fig. 8. In [7] the area of such hysteresis loops have been taken into account in order to evaluate energetic properties of the device. At this point, we have to notice that a pinched hysteresis loop is a fingerprint of memelements but it is not a unique characterization of such elements since the shape changes depending on the excitation. Therefore,
energetic statements based on the hysteresis loop for a static memcapacitive system are valid only for the particularly investigated input. In contrast to that, an energetic definition leads to universally valid statements independent of the exploited model, parameters, and the excitation.

C. Differential Memcapacitive System

The remaining differential definition of memcapacitive devices (1b) yields to the current

\[ i_d = i_e - G u. \]  

(8)

Up to the sign for the conductance, the differential definition leads to a similar modeling approach as the static memcapacitive system, cf. Fig. 9. Consequently, the insights regarding passivity of static memcapacitive devices are also valid for a differential definition.

As a result, we have to keep in mind that a consistent modeling of memcapacitive devices is not trivial. From a circuit theoretical point of view, modeling of such devices in an energetic matter instead of an approach based on static memcapacitive systems, which is more common in the literature, is worthwhile. Especially for complex self-organizing systems, e.g. neuromorphic circuits, a consistent modeling can help for investigations regarding the stability of the overall system. For emulation purposes, a deeper but also a consistent modeling approach is of importance.

It is noticeable that the ideal transformer is energy neutral independent of the transformation ratio. In consequence, these theoretical considerations are valid for arbitrary memcapacitive models, which makes the presented approach very powerful and general. We do not have to make an effort about limitations and restrictions with respect to model parameters or excitation waveforms in order to avoid an active behavior of the device.

IV. CONCLUSION

In this work, a consistent modeling of memcapacitive systems has been introduced. Important energetic properties of the analog device have been taken into account during this modeling procedure. In contrast to common memcapacitive models in the state of the art, the passivity of the device can be ensured in the approach represented here. Therefore, an energetic definition of nonlinear capacitors based on circuit theoretic considerations has been utilized.

As it turned out, an energetic definition of memcapacitive systems leads to necessary as well as sufficient conditions in order to obtain passivity. A corresponding electrical circuit of an energetic memcapacitive system based on a parametric representation of an ideal transformer has been proposed. Since the ideal transformer is energy neutral independent of its transmission ratio, the energetic memcapacitive is not only passive but also lossless. The presented approach is a general modeling solution and is not limited to a particular model or to a special excitation and can be utilized in several applications.

Memcapacitive devices offer novel possibilities in different areas, like robotics, neuromorphic circuits, and self-organizing networks. Especially the latter needs a consistent model because of stability investigations. Regarding a real-time capable implementation of memcapacitive systems hardware emulators based on electrical models are needed. In this context, emulators which are closer to real devices can be developed by utilizing the consistent modeling procedure.

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