Mimicking Gait Pattern Generators

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Abstract—To replicate the behavior of (large scale) biologically inspired networks through neuromorphic circuits the question arises on how to model the synaptic interconnections between neurons. Biological representations often use unidirectional couplings between neurons which waive problems on electrical models, as electrical networks in general are not free from feedback. We propose a simple solution based on a four port circulator to overcome this issue. The method involves attaching appropriate resistances to this circulator, depending on the desired coupling. The wave digital representation of this approach reveals that our modeling enables unidirectional and free from feedback interconnections between neurons. We use the proposed method to investigate multiple gait pattern generators of a four-legged animal.

I. INTRODUCTION

The ability to memorize and learn has been affiliated to the interactions between millions of neurons which are connected through synapses in the brain. Neurons are often modeled as oscillators, such as the FitzHugh-Nagumo [1], Hodgkin-Huxley [2] and the Morris-Lecar [3] models. The latter is a popular model also in recent publications in the field of computational neuroscience [4], because its differential equations for the channel states are only of first order. A parameter-optimized version of the model was currently proposed [5].

Learning behavior is abstractly often associated with neurons whose membrane potentials are synchronized [6]–[9]. To study such synchronization behavior, synapses play an essential part in the investigations of coupled neurons. A fundamental role in this complex network structure has been assigned to the specific strengths of the synaptic interconnections, which determine the respective influence between coupled neurons. Essentially, there are two types of couplings, namely the chemical and the electrical coupling. The basic idea of a chemical synapse is to represent the effect of neurotransmitters and their consequential time delay. Chemically coupled neurons have been investigated, such as synchronization and bifurcations in inhibitory chemically coupled Morris-Lecar neurons [6], [7].

In electrical synapses, the modeling focuses on the difference between presynaptic and postsynaptic membrane potential. Existing publications have examined, e.g., the synchronization of coupled Morris-Lecar neurons with an electrical synapse [9] as well as the modeling and implementation of two Hodgkin-Huxley neurons with electrical coupling [8]. Since electrical synapses are important in case of synchronizing the electrical activity of groups of neurons [10], our work focuses on this type of synapse. We use these insights to generate a simple gait pattern generator, which generally are subject to current research, e.g. [11].

The paper is organized as follows. In Sec. II, the modeling of the synapse is explained and its wave digital form is provided. Furthermore we elaborate on how these synapses are attached to neurons in the electrical and the wave digital representation of their electric circuits. In Sec. III, we associate the four legs of a dog with four synaptically coupled neurons and mimic different gait patterns by adjusting only the coupling strengths of the synapses. It is shown that a smooth transition from one gait pattern to another is possible, even if the couplings change abruptly. A final conclusion summarizes the main results.

II. SYNAPSE MODEL AND WAVE DIGITAL REPRESENTATION

According to [8], [9], the coupling of an electrical synapse can be modeled as a simple constant conductance with its value denoting the coupling strength. This assumption is supported by an equivalent circuit of two coupled cells, where the coupling is a single conductance [12]. However, when connecting two neurons with only a resistance in series, there is no freedom from feedback. Hence, this indicates that the self-coupling which is considered to be zero by [8] is in fact not zero. Consequently, an alternative approach based on a circulator is introduced in this work.

A. Graph Representation

In biology, a graph representation is often used to describe coupled neurons like in Fig. 1 [13]. However, it remains unclear how the equivalent electrical circuit is to be designed. Especially unidirectional couplings waive problems, since electrical networks in general are not free from feedback. We show that using a wave digital description of the problem yields to a simple solution that enables all possible coupling scenarios.
The incident waves \( a_k \) and the reflected waves \( b_k \) are related to current and voltage through the bilinear transformation

\[
\begin{bmatrix}
a_k \\
b_k
\end{bmatrix} = \begin{bmatrix}
1 & R \\
1 & -R
\end{bmatrix} \begin{bmatrix}
u_k \\
i_k
\end{bmatrix}, \quad \text{with } R > 0, \ k \in \{1,2\},
\]

(2)

where \( R \) is an arbitrarily chosen positive constant that is associated with the port. It can be observed that the incident waves of one port equal the reflected waves of the other port:

\[
\begin{bmatrix}
a_1 \\
b_1
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix} \begin{bmatrix}
a_2 \\
b_2
\end{bmatrix},
\]

(3)

To investigate the power flows in Fig. 2, it holds that for port \( k \in \{1,2\} \)

\[
\begin{align*}
p_{k,\text{in}} &= \frac{a_k^2}{4R}, & p_{k,\text{out}} &= \frac{b_k^2}{4R}, \\
p_k &= p_{k,\text{in}} - p_{k,\text{out}}.
\end{align*}
\]

(4a)

(4b)

By (3) and (4a),

\[
\begin{bmatrix}
p_{1,\text{in}} \\
p_{1,\text{out}}
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix} \begin{bmatrix}
p_{2,\text{in}} \\
p_{2,\text{out}}
\end{bmatrix},
\]

(5)

To ensure freedom from feedback and especially prevent self-coupling, we propose the use of a four-port circulator with two attached resistances as shown in Fig. 3. This model of a synaptic interconnection between neurons is comparably simple to a coupling implemented by only a constant resistance [8], but has the additional benefit of being easily configurable for the desired application while being free from feedback. In particular, its scattering parameters representation is lucid. Translating the proposed setup into the wave digital domain, the two resistances result in two reflection coefficients \( \rho_1 \) and \( \rho_2 \). In general, by (5), it holds that

\[
\begin{bmatrix}
p_{1,\text{in}} \\
p_{1,\text{out}}
\end{bmatrix} = \begin{bmatrix}
0 & \rho_2^2 \\
\rho_1^2 & 0
\end{bmatrix} \begin{bmatrix}
p_{2,\text{in}} \\
p_{2,\text{out}}
\end{bmatrix},
\]

(6a)

with

\[
\rho_k = \frac{R_k - R}{R_k + R}, \ k \in \{1,2\}.
\]

(6b)

It can be seen from (6a) that the outgoing power flow of one port only depends on the incoming power flow of the other port. Hence, the power flow from one neuron to another is unidirectionally and immediately controllable. Depending on the desired functionality, the choice of \( R_k \) determines \( \rho_k \), whereas \( \rho_k \in [-1,1] \) by (6b). Here, the special cases

\[
R_k \to \infty \Rightarrow \rho_k = 1,
\]

\[
R_k \to 0 \Rightarrow \rho_k = -1,
\]

\[
R_k = R \Rightarrow \rho_k = 0,
\]

enable anti-cyclical coupling, cyclical coupling and complete decoupling, respectively. It should also be noted that in general all coupling strengths are possible, yet in the context of animal gait pattern generators only these special cases are relevant. The direct interconnection of two ports like the one shown in Fig. 2 is only possible, if their port resistances are equal. If they do not match, one utilizes adaptors to overcome this issue.
C. Connecting Multiple Neurons

The essential task of the axon hillock is to sum up all the input signals. The most established neuron models like the Hodgkin-Huxley, FitzHugh-Nagumo and Morris-Lecar model all assume a current as an input signal. To electrically represent the functionality of the axon hillock, a parallel interconnection network is suitable. Assuming a neuron has \( n - 1 \) other neurons attached to it, a \( n \)-port parallel adaptor is required in the wave digital domain. This is because using a parallel adaptor will make the resulting power flow the difference between all ingoing and all outgoing power flows. A \( n \)-port parallel adaptor can be implemented connecting multiple three-port parallel adaptors, shown in Fig. 4, where each of these can be described through

\[
\begin{bmatrix}
    b_1 \\
    b_2 \\
    b_3
\end{bmatrix} =
\begin{bmatrix}
    \gamma - 1 & 1 - \gamma & -1 \\
    \gamma & -\gamma & 1 \\
    \gamma & 1 - \gamma & 0
\end{bmatrix}
\begin{bmatrix}
    a_1 \\
    a_2 \\
    a_3
\end{bmatrix},
\]

(7a)

with

\[
\gamma = \frac{R_2}{R_1 + R_2}.
\]

(7b)

This decomposition yields to only one adaptor coefficient \( \gamma \) per adaptor, which are \( n - 2 \) in total. Multiple neurons, as depicted in Fig. 1, can then be coupled with each other by attaching a parallel adaptor to them and then connect them via the circulator of Fig. 3, as shown in Fig. 5. It becomes evident that the transfer from the graph depiction into the electrical/wave digital domain was not straightforward and required careful considerations.

III. Animal Gait Pattern Generator

In the following we will study fundamental setups of four interconnected neurons. Our aim is to mimic the gait patterns running, pacing and trotting of a dog by only changing couplings strengths between neurons. All three gait patterns have in common that each leg can be classified into one of two sets \( S_1 \) and \( S_2 \), where all intra-set legs move cyclical, and all inter-set legs move anti-cyclical to one another.

<table>
<thead>
<tr>
<th>Gait Pattern</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>running</td>
<td>front left</td>
<td>back left</td>
</tr>
<tr>
<td></td>
<td>front right</td>
<td>back right</td>
</tr>
<tr>
<td>pacing</td>
<td>front left</td>
<td>front right</td>
</tr>
<tr>
<td></td>
<td>back left</td>
<td>back right</td>
</tr>
<tr>
<td>trotting</td>
<td>front left</td>
<td>front right</td>
</tr>
<tr>
<td></td>
<td>back right</td>
<td>back left</td>
</tr>
</tbody>
</table>

The investigated gait patterns are conceptually shown in Fig. 6. We associate each of these legs of a dog with a neuron.

For our simulations we deploy the simplified Morris-Lecar neuron model [3], [4] with an optimized set of parameters that recently has been shown to achieve a more biologically inspired membrane potential behavior [5]. The simulation results are shown in Fig. 7 and are discussed in the following in detail.

First, to simulate a running dog, the synaptic couplings between the neurons are chosen as shown in the left graph of Fig. 6. In Fig. 7 we let the membrane potentials of the neurons start at different initial conditions, yet they achieve their desired trajectories quickly after just two oscillations, following indeed a running dog’s gait pattern. The shape of the first oscillation of each neuron is explained by their transient behavior.

At time \( t = T_1 \) the coupling weights change abruptly so that the behavior of a pacing dog is achieved, see middle graph of Fig. 6. Although the switching is abrupt, we can observe a smooth transition behavior from running to pacing. It is notable that hardly any changes in the trajectories of the membrane potential are visible during the first four oscillations after the switching moment \( T_1 \). After that, between five and six oscillations are required to transition from running gait pattern to pacing gait pattern. During the transition phase it can be observed that the amplitudes of the membrane potentials temporarily become smaller. This is because the underlying
synaptic interconnection of the neurons do not match their relative behavior. For that reason they interact destructively until the membrane potentials change according to their synaptic coupling. Therefore, their membrane potentials’ amplitudes are the strongest when the transition phase is successfully completed.

Once the gait pattern of pacing is achieved, the coupling weights are again changed abruptly to those of a trotting dog at \( t = T_2 \) as depicted in the right graph of Fig. 6. Similarly to the first switching moment \( T_1 \) we observe two events. First, the relative membrane potential behavior does not match the interconnection of the neurons, which is why the amplitudes of their membrane potentials are reduced. Second, during the first 6 oscillations, no change in the relative trajectories can be seen. After another 4 oscillations, the transition phase is completed and the desired behavior of a trotting dog can be observed while the full amplitudes of the membrane potential are restored. Again, the transitions was smoothly, although the change in coupling weights was abrupt.

At \( t = T_3 \), we switch off the neurons’ applied current to simulate a standstill. This is successfully obtained, as can be seen in Fig. 7.

IV. Conclusion

In this work, we have considered neuronal oscillators which are interconnected by synaptic couplings. State of the art electrical representations of synapse models lack the ability to incorporate unidirectional couplings, because they are generally not free from feedback. Our approach involves the use of a four-port circulator to overcome this problem. It has been shown that all desirable coupling strengths are possible and we have explicitly discussed the special cases of cyclical and anti-cyclical coupling, as well as complete decoupling. Unidirectional couplings between neurons are also achievable through our solution. The transition into the wave digital representation enables a comprehensive and easy to configure synaptic coupling for neurons. We have also explained in detail how to transfer from a graph representation in biology to an electrical/wave digital representation. Finally, we have investigated four coupled Morris-Lecar neurons to mimic three different gait patterns of a dog. By associating each neuron to one foot of a dog, we have demonstrated the correct working of our synapse model, including the detailed description of the transition phases.

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REFERENCES