Power Efficiency of Improper Signaling in MIMO Full-Duplex Relaying for K-User Interference Networks

Ali Kariminezhad, Aydin Sezgin and Marius Pesavento

Abstract—**Multi-hop communication is a spectral-efficient approach for connecting multiple pairs when direct links are absent or have insufficient strength. In this paper, we highlight the benefits of improper Gaussian signaling for a multi-pair full-duplex MIMO relay network in terms of power efficiency. By employing improper Gaussian transmission, the power minimization problem under rate constraints is intrinsically a non-convex problem due to non-convex rate expressions in the constraint set. We utilize Fenchel's inequality to linearize the non-convex part of the constraint, which results in feasible solutions of the problem in polynomial time. This approximation results in an upper-bound for the original problem which gets tighter in the number of iterations performed. The solution is compared with analytical beamforming solutions, namely zero-forcing (ZF) and maximum-ratio transmission/combining (MRT/MRC). Due to the feasibility of single rank real-valued transmission in improper Gaussian signaling, the optimal solution switches to single rank transmission depending on the constraints defined by the requests of the users, which can not be captured by proper Gaussian signaling.**

I. Introduction

Future communication networks will benefit exploiting efficient relays equipped with multiple antennas in order to fulfil the rate demands of the crowd of users with weak direct links. This exploitation of spatial domain for communication is due to the scarcity of time and frequency resources for achieving ever increasing quality of service (QoS) demands of the users.

Another technique to satisfy these demands is to transmit and receive at the same resource instants i.e., time and frequency, by full-duplex communication, [1]. Thereby, the spectral efficiency can be almost doubled compared to half-duplex operation [2]. This improvement in the spectral efficiency does not come for free and requires sophisticated processing at the full-duplex transceivers. The most significant hurdle in full-duplex communication is the self-interference (SI) cancellation. The authors in [3], [4] propose transmitter and receiver isolation for reducing the projection of transmit signal at the receiver, while [5]–[7] consider analog and digital signal processing tasks to cope with this impairment. The authors in [8] perform SI channel estimation, by which SI can be actively suppressed in the digital domain. These types of cancellation methods are mainly developed for single-input single-output (SISO) systems. Employing multiple antennas for transmission and reception provides the opportunity for spatial domain SI cancellation based on the SI channel knowledge [9]–[12]. This type of cancellation can be realized by steering the transmit signal onto the null-space of the SI channel. In practice, taking into account imperfect SI channel estimation and the aforementioned cancellation methods, residual self-interference (RSI) remains which is destructive and limits the achievable information rate. The portion of RSI due to the imperfect channel estimation can be arbitrarily decreased depending on the pilot sequence length in the SI channel estimation phase [6]. However, transmitter noise is the remaining RSI that impairs the system performance [13]. Transmitter noise is mainly due to the imperfect transmitter operations such as non-linearity of the transmit power amplifiers, limited dynamic range etc., that deteriorate the transmit signal at the transmission phase [13]–[15].

In this paper we do not ignore RSI in the signal generation phase for optimal Gaussian signaling design. Furthermore, we consider general Gaussian transmission known as improper Gaussian signaling in our design. Improper Gaussian signaling has been shown to improve the degrees-of-freedom (DoF) of interference networks [16], [17]. Furthermore, information rate improvement by utilizing improper Gaussian signaling in a two-user interference channel is investigated by [18], [19]. Considering improper Gaussian signaling in a multi-pair relay network with interfering links, we investigate the power minimization problem under achievable information rate constraints. The problem turns out to be a non-convex problem due to non-convex constraint set imposed from the difference of two concave functions (DC program) [20]. The authors in [21] utilize successive convex inner approximations based on linearization of the non-convex part of the constraint, while [22] uses successive inner convex approximation as second-order cone programs to obtain feasible (generally suboptimal) solution. In this paper, we exploit a convex inner-approximation on the non-convex constraint set by utilizing the concept of conjugate function. The obtained problem turns to be a semidefinite program which can be solved by interior-point methods in polynomial time. By successive inner approximations on the constraint set, the gap between the inner-approximated and the non-convex constraint sets is resolved in an iterative manner.

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II. SYSTEM MODEL

We consider a full-duplex relay interference channel, where the relay is equipped with N_t and N_r transmit and receive antennas, respectively. As shown in Fig. 1, K sources are linked with their corresponding destinations through this relay by strong channels while the direct links are weak. The channel input-output relation between kth source and the relay and between the relay and the kth destination is written as

$$\mathbf{y}_\mathrm{r} = \sum_{k=1}^{K} \mathbf{h}_{s_k\mathrm{r}}(x_k + e_k) + \mathbf{H}_\mathrm{rr}(\mathbf{x}_\mathrm{r} + \mathbf{e}_\mathrm{r}) + \mathbf{n}_\mathrm{r}, \tag{1}$$

$$y_k = \mathbf{h}_{\mathrm{r}d_k}^H(\mathbf{x}_\mathrm{r} + \mathbf{e}_\mathrm{r}) + \sum_{l=1}^{K} h_{s_l d_k}(x_l + e_l) + n_{d_k}, \tag{2}$$

respectively. Here $\mathbf{h}_{s_k\mathrm{r}}$, $\mathbf{h}_{\mathrm{r}d_k}$, $h_{s_l d_k}$ and $\mathbf{H}_\mathrm{rr}$ are the source-relay, relay-destination, source-destination, and relay self-interference (SI) channels, respectively. These channels are assumed to be globally known. Transmitter noise at the kth source and the relay are represented by e_k and $\mathbf{e}_\mathrm{r}$, respectively. Moreover, the receiver noise realizations at the relay and destination k are represented by $\mathbf{n}_\mathrm{r}$ and n_{d_k}, respectively.
In this paper we ignore transmitter noise from the sources toward the relay and from the relay toward the destinations. However, transmitter noise on the SI channel is not ignored. This assumption is valid due to comparably large strength of the SI channel compared to source-relay and relay-destination channels. Having this assumption, the channel input-output relationship reduces to

$$\mathbf{y}_\mathrm{r} = \sum_{k=1}^{K} \mathbf{h}_{s_k\mathrm{r}} x_k + \mathbf{H}_\mathrm{rr}(\mathbf{x}_\mathrm{r} + \mathbf{e}_\mathrm{r}) + \mathbf{n}_\mathrm{r}, \tag{3}$$

$$y_k = \mathbf{h}_{\mathrm{r}d_k}^H \mathbf{x}_\mathrm{r} + \sum_{l=1}^{K} h_{s_l d_k} x_l + n_{d_k}. \tag{4}$$

The full-duplex relay applied spatial post-processing to the signals received from multiple antennas. After decoding, the relay re-encodes the messages to obtain transmit symbols. These symbols are then precoded by the relay and further forwarded toward the corresponding destinations.

A. Self-Interference Channel

We assume that a sufficiently long channel estimation phase is used so that the estimation errors are negligible. Hence, by the SI channel knowledge $\mathbf{H}_\mathrm{rr}$ we cancel a portion of transmit signal as

$$\begin{aligned}\hat{\mathbf{y}}_\mathrm{r} =& \mathbf{y}_\mathrm{r} - \mathbf{H}_\mathrm{rr}\mathbf{x}_\mathrm{r} \\ =& \sum_{k=1}^{K} \mathbf{h}_{s_k\mathrm{r}} x_k + \mathbf{H}_\mathrm{rr}\mathbf{e}_\mathrm{r} + \mathbf{n}_\mathrm{r},\end{aligned} \tag{5}$$

which consists of the sources transmit signal and relay transmitter noise which is modelled in the next section.

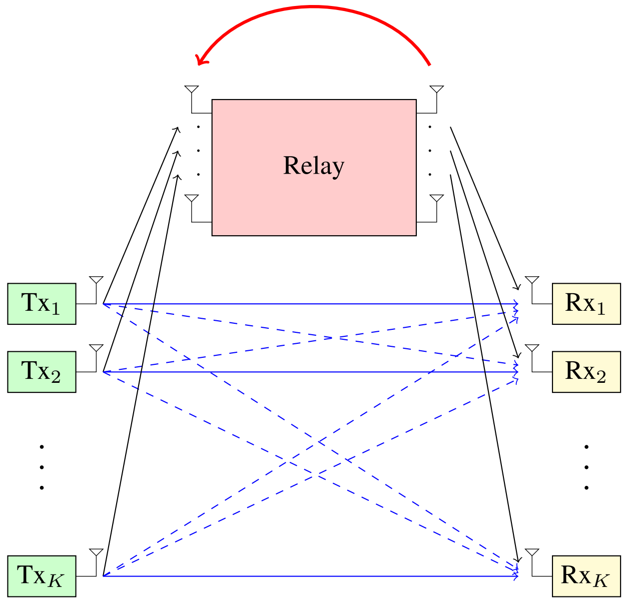


Fig. 1. Relay interference channel with full-duplex operation. The relay is equipped with multiple transmitter and receiver antennas while the source and destination nodes have a single antenna. Inter-user interference is shown by dashed lines, while the self-interference channel is shown in red.

B. Transmitter Noise

Transmitter noise mainly accounts for the limited dynamic range (DR) of the transmitter, the I/Q modulation imperfections, and the non-linear behaviour of power amplifiers. In this paper, we model the transmitter noise to be statistically independent from the transmit signal at the relay as, [6], [23],

$$\mathbf{e}_\mathrm{r} \perp\!\!\!\perp \mathbf{x}_\mathrm{r}, \tag{6}$$

$$\mathbf{e}_\mathrm{r} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_e) \tag{7}$$

$$\mathbf{Q}_e = \kappa \mathrm{diag}(\mathbf{Q}_\mathrm{r}), \text{ where } \kappa \ll 1. \tag{8}$$

Assumption (6) represents the statistical independences between the transmitter noise and the transmit signal, while the transmitter noise is generally modelled to follow a zero-mean Gaussian distribution with covariance $\mathbf{Q}_e$, given by (7). Considering proper Gaussian transmission, the transmitter noise follows a symmetric Gaussian distribution which is the essence of the model in [6]. Otherwise, the transmitter noise is asymmetrically Gaussian distributed which is the focus of this paper. Eq. (8) depicts the second-order statistical relation between the relay transmit signal and the transmitter noise which is approved by the hardware setup in [24], [25].

C. Improper Gaussian Signaling

We allow improper Gaussian signaling at the sources and the relay. By widely linear precoding, we access the real-valued components of a complex Gaussian random variable. This provides the opportunity for non-equal power allocation for the real and imaginary components of the complex-valued signals. Besides, an extra degrees-of-freedom (DoF) is accessible by allowing correlation between the real and imaginary components. This type of Gaussian signaling is well-known as improper Gaussian signaling which removes the symmetry assumption from the Gaussian random signal, cf. Appendix A. In order to capture the properness of the Gaussian signal we consider the real-valued representation. Here, we cast the real-valued representation of the complex-valued system model of (1) as

$$\mathbf{z}_{\mathrm{r}} = \sum_{k=1}^{K} \mathbf{G}_{s_k\mathrm{r}}\hat{\mathbf{x}}_k + \mathbf{G}_{\mathrm{rr}}\hat{\mathbf{e}}_{\mathrm{r}} + \hat{\mathbf{n}}_{\mathrm{r}}, \tag{9}$$

$$\mathbf{z}_k = \mathbf{G}_{\mathrm{r}d_k}\hat{\mathbf{x}}_{\mathrm{r}} + \sum_{l=1}^{K} \mathbf{G}_{s_l d_k}\hat{\mathbf{x}}_l + \hat{\mathbf{n}}_{d_k}, \tag{10}$$

where $\mathbf{z}_r \in \mathbb{R}^{2N_{\mathrm{r}}}$ and $\mathbf{z}_k \in \mathbb{R}^2$ are the real-valued vectors that are obtained by stacking the real and imaginary components of the complex-valued received signal vector at the relay and destinations. Moreover, the real and imaginary components of the complex-valued transmit signal from the sources and the relay are stacked into a vector and are represented by $\hat{\mathbf{x}}_k \in \mathbb{R}^2$ and $\hat{\mathbf{x}}_{\mathrm{r}} \in \mathbb{R}^{2N_{\mathrm{t}}}$. The transmitter noise from the relay output to its input is given by $\hat{\mathbf{e}}_{\mathrm{r}} \in \mathbb{R}^{2N_{\mathrm{t}}}$. It is important to note that the receiver noise at the relay and at the destinations are represented in the real domain as well. Correspondingly, the complex-valued vector channels are expressed as matrices in order to provide consistency with the complex-valued system model, cf. Appendix B.

III. Achievable Rates

We define the achievable rate from source k to the relay and from the relay to destination k as $R_{s_k\mathrm{r}}$ and $R_{\mathrm{r}d_k}$, respectively. Then the achievable rate from source k to destination k is defined as $R_{s_kd_k}$ which can be formulated as

$$R_{s_kd_k} = \min\{R_{s_k\mathrm{r}}, R_{\mathrm{r}d_k}\}. \tag{11}$$

By considering the real-valued system model of (9) and (10) and treating interference as noise (TIN) at the relay and the destinations, we can achieve the information rates as stated on the top of the next page [26]. These rates are achievable by assuming Gaussian codebooks at the transmitters, i.e., the sources and the relay. In (12) and (13), the transmit covariance matrices from the kth sources and the relay are represented by $\hat{\mathbf{Q}}_k$ and $\hat{\mathbf{Q}}_{\mathrm{r}}$, respectively. Note that, the symmetric positive-semi-definite real-valued covariance matrices $\hat{\mathbf{Q}}_k$ and $\hat{\mathbf{Q}}_{\mathrm{r}}$ are of dimensions 2×2 and $2N_{\mathrm{t}} \times 2N_{\mathrm{t}}$, respectively. Furthermore, the AWGN variance at each receive antenna (either at the relay or at the destination) is denoted by σ^2. Due to linear precoding we formulate the transmit covariance matrix at the relay as

$$\hat{\mathbf{Q}}_{\mathrm{r}} = \sum_{k=1}^{K} \hat{\mathbf{Q}}_{\mathrm{r}_k}, \tag{14}$$

where $\hat{\mathbf{Q}}_{\mathrm{r}_k}$ is the transmit covariance matrix from the relay to the kth destination. Note that in (12) and (13), the transmit power is embedded into the transmit covariance matrices. Hence, the power constraint at the sources and relay can be expressed as,

$$\mathrm{tr}(\hat{\mathbf{Q}}_k) \leq P_{k_{max}}, \tag{15}$$

$$\mathrm{tr}(\hat{\mathbf{Q}}_{\mathrm{r}}) \leq P_{\mathrm{r}_{max}}, \tag{16}$$

where maximum power budgets at source k and the relay are represented by $P_{k_{max}}$ and $P_{\mathrm{r}_{max}}$, respectively.

IV. Power Minimization

In this section we study the power minimization problem for the considered setup. The sum-power minimization is subjected to the quality of service (QoS) demands of the communicating pairs. This quality of service demands are reflected by the information rate constraints. We formulate the problem as

$$\min_{\hat{\mathbf{Q}}_{\mathrm{r}}\in\mathbb{S}^{2N_t},\ \hat{\mathbf{Q}}_k\in\mathbb{S}^2} \quad \mathrm{tr}(\hat{\mathbf{Q}}_{\mathrm{r}}) + \sum_{k=1}^{K}\mathrm{tr}(\hat{\mathbf{Q}}_k) \tag{17}$$

$$\text{subject to} \quad R_{s_kd_k} \geq \psi_k,\ \forall k \in \{1,...,K\}, \tag{17a}$$

$$\hat{\mathbf{Q}}_{\mathrm{r}} \succeq 0, \tag{17b}$$

$$\hat{\mathbf{Q}}_k \succeq 0,\ \forall k \in \{1,...,K\}, \tag{17c}$$

where the rate demands are given by (17a). Note that ψ_k is the rate demand for the kth communicating pair. The constraints (17b) and (17c) guarantee the solution to be positive semidefinite. Note that, the transmit covariance matrices at the relay and kth source are constrained to belong to the set of symmetric matrices, i.e., $\hat{\mathbf{Q}}_{\mathrm{r}} \in \mathbb{S}^{2N_{\mathrm{t}}}$ and $\hat{\mathbf{Q}}_k \in \mathbb{S}^2$. By merging (11)-(13) into the rate constraints in (17a), problem (17) can be rewritten as

$$\min_{\hat{\mathbf{Q}}_{\mathrm{r}}\in\mathbb{S}^{2N_{\mathrm{t}}},\ \hat{\mathbf{Q}}_k\in\mathbb{S}^2} \quad \mathrm{tr}(\hat{\mathbf{Q}}_{\mathrm{r}}) + \sum_{k=1}^{K}\mathrm{tr}(\hat{\mathbf{Q}}_k) \tag{18}$$

$$\text{subject to} \quad R_{s_k\mathrm{r}}\left(\hat{\mathbf{Q}}_{\mathrm{r}}, \hat{\mathbf{Q}}_k\right) \geq \psi_k,\ \forall k, \tag{18a}$$

$$R_{\mathrm{r}d_k}\left(\hat{\mathbf{Q}}_{\mathrm{r}}, \hat{\mathbf{Q}}_k\right) \geq \psi_k,\ \forall k, \tag{18b}$$

$$\hat{\mathbf{Q}}_{\mathrm{r}} \succeq 0, \tag{18c}$$

$$\hat{\mathbf{Q}}_k \succeq 0,\ \forall k, \tag{18d}$$

where the source-relay and relay-destination achievable rates are the functions of transmit covariance matrices as in (12) and (13). Optimization problem (18) consists of a semidefinite program (SDP) with a linear objective function and a non-convex constraint set. The non-convexity is due to the rate constraints in (18a) and (18b), since the achievable rates in (12) and (13) are the difference of concave functions. Thus, these constraints are neither convex nor concave in general and they render the problem to a non-convex problem. Here, we linearize the second concave function in the rate expressions by using Fenchel's inequality which is based on the concept of conjugate function [27]. Then, the resultant convex-constraint semidefinite program (SDP) can be solved by interior point methods efficiently.

The second concave terms in (12) and (13) can be upper-bounded using the following lemma which is a direct implication from the section 3.3 of [27], [cf. equation 3.18].

Lemma 1: For given $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{a\times b}$, the function $\log|\mathbf{AXA}^T + \mathbf{BYB}^T + \mathbf{I}_a|$ is upper-bounded by a linear

$$R_{s_k\mathrm{r}} = \frac{1}{2}\log|\sum_{j=1}^{K}\mathbf{G}_{s_j\mathrm{r}}\hat{\mathbf{Q}}_j\mathbf{G}_{s_j\mathrm{r}}^H + \mathbf{G}_{\mathrm{rr}}\hat{\mathbf{Q}}_e\mathbf{G}_{\mathrm{rr}}^H + \sigma^2\mathbf{I}_{N_\mathrm{t}}| - \frac{1}{2}\log|\sum_{\substack{j=1\\ j\neq k}}^{K}\mathbf{G}_{s_j\mathrm{r}}\hat{\mathbf{Q}}_j\mathbf{G}_{s_j\mathrm{r}}^H + \mathbf{G}_{\mathrm{rr}}\hat{\mathbf{Q}}_e\mathbf{G}_{\mathrm{rr}} + \sigma^2\mathbf{I}_{N_\mathrm{t}}| \quad (12)$$

$$R_{\mathrm{r}d_k} = \frac{1}{2}\log|\mathbf{G}_{\mathrm{r}d_k}\hat{\mathbf{Q}}_\mathrm{r}\mathbf{G}_{\mathrm{r}d_k}^H + \sum_{j=1}^{K}\mathbf{G}_{s_jd_k}\hat{\mathbf{Q}}_j\mathbf{G}_{s_jd_k}^H + \sigma^2\mathbf{I}_2| - \frac{1}{2}\log|\mathbf{G}_{\mathrm{r}d_k}\sum_{\substack{j=1\\ j\neq k}}^{K}\hat{\mathbf{Q}}_{\mathrm{r}_j}\mathbf{G}_{\mathrm{r}d_k}^H + \sum_{j=1}^{K}\mathbf{G}_{s_jd_k}\hat{\mathbf{Q}}_j\mathbf{G}_{s_jd_k}^H + \sigma^2\mathbf{I}_2| \quad (13)$$

function in $\mathbf{X}$, $\mathbf{Y}$ as

$$\log|\mathbf{AXA}^T + \mathbf{BYB}^T + \mathbf{I}_a| \leq \log|\mathbf{\Gamma}| + \mathrm{Tr}(\mathbf{\Gamma}^{-1}(\mathbf{AXA}^T + \mathbf{BYB}^T + \mathbf{I}_a)) - \mathrm{Tr}(\mathbf{I}_a), \quad (19)$$

for all $\mathbf{\Gamma} \in \mathbb{R}^{a\times a}$ so that $\mathbf{\Gamma} \succeq \mathbf{0}$. Equality holds when $\mathbf{\Gamma} = \mathbf{AXA}^T + \mathbf{BYB}^T + \mathbf{I}_a$.

Proof- cf. Appendix C.

By applying Lemma 1, we linearize the second terms in the rate expressions in (12) and (13) at the cost of adding extra variables. Since the second log expressions (concave function) in the rate constraint are upper-bounded, the problem might turn out to be infeasible for any given arbitrary $\mathbf{\Gamma}_{s_k\mathrm{r}}$ and $\mathbf{\Gamma}_{\mathrm{r}d_k}$, although the original problem is feasible. This is due to the adjusting role of $\mathbf{\Gamma}_{s_k\mathrm{r}}$ and $\mathbf{\Gamma}_{\mathrm{r}d_k}$ in the feasibility of the interior. Hence, primitively by initializing $\mathbf{\Gamma}_{s_k\mathrm{r}}$ and $\mathbf{\Gamma}_{\mathrm{r}d_k}$ we examine the feasibility check problem. Besides, we define the virtual noise variance which is λ times smaller than the actual noise variance, where $\lambda \gg 1$ [28]. This way, we enlarge the domain of the constraint set for arbitrary positive semidefinite $\mathbf{\Gamma}_{s_k\mathrm{r}}$ and $\mathbf{\Gamma}_{\mathrm{r}d_k}$. It is important to note that, this virtual noise variance is eventually considered in the solution of the optimization problem. The quality of the optimization is further improved by re-initializing $\mathbf{\Gamma}_{s_k\mathrm{r}}$ and $\mathbf{\Gamma}_{\mathrm{r}d_k}$ in an iterative manner based on the solutions of the previous iteration which is mainly a successive inner approximation [22], [29], [30]. Mathematically,

$$\log|\mathbf{AXA}^T + \mathbf{BYB}^T + \mathbf{I}_a| = \log|\mathbf{\Gamma}| + \mathrm{Tr}(\mathbf{\Gamma}^{-1}(\mathbf{AXA}^T + \mathbf{BYB}^T + \mathbf{I}_a)) - \mathrm{Tr}(\mathbf{I}_a) + \epsilon, \quad (20)$$

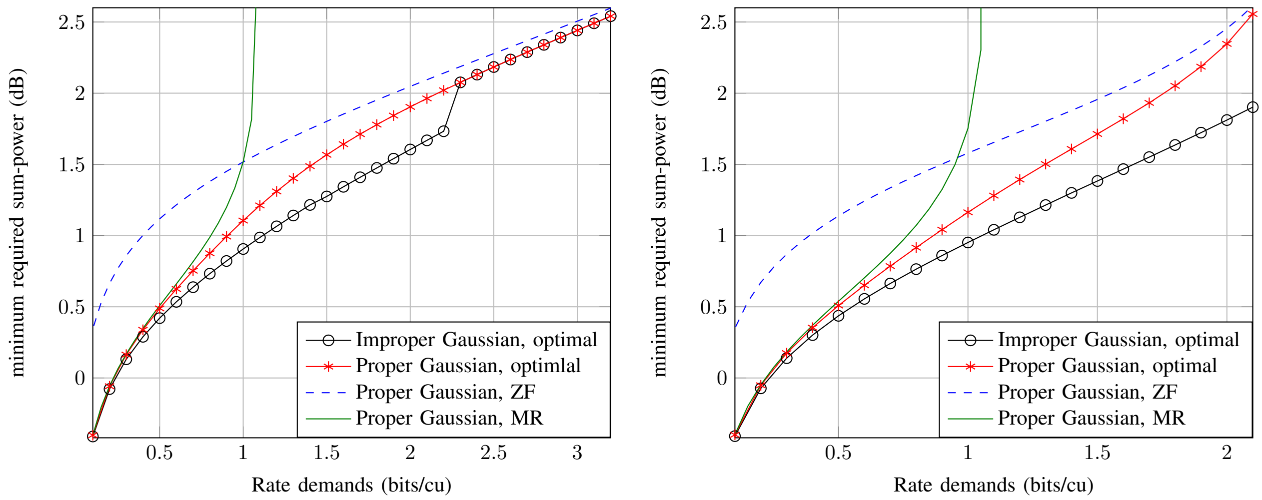
where $\epsilon \geq 0$ is the gap between the upper-bound and the actual value of the second log expressions in the rate constraints. Note that, $\epsilon \rightarrow 0$ by iterations over $\mathbf{\Gamma}_{s_k\mathrm{r}}$ and $\mathbf{\Gamma}_{\mathrm{r}d_k}$. Algorithm 1 explains the procedure elaborately.

Algorithm 1 Power minimization under rate constraints

1: $\sigma^2 \leftarrow \sigma^2/\lambda,\ \forall\lambda \gg 1$
2: $t = 1$
3: Linearize the second log terms in the rate expressions
4: Initialize $\mathbf{\Gamma}_{s_k\mathrm{r}}^{(1)}, \mathbf{\Gamma}_{\mathrm{r}d_k}^{(1)},\ \forall k$ and solve the convex feasibility check problem to ensure non-empty interior.
5: Solve the semidefinite program (18) $\rightarrow$ $\hat{\mathbf{Q}}_\mathrm{r}^{(t)}$ and $\hat{\mathbf{Q}}_k^{(t)},\ \forall k$
6: Calculate the gap $\epsilon^{(t)}$ from (20)
7: Determine the resolution of the solution, e.g. ϵ^*
8: **while** $\epsilon^{(t)} \geq \epsilon^*$ **do**
9: Calculate optimal $\mathbf{\Gamma}_{s_kr}^{(t)}, \mathbf{\Gamma}_{rd_k}^{(t)}$ from the solution acquired from step 6, according to lemma 1.
10: $t = t + 1$
11: Solve the SDP (18) to acquire $\hat{\mathbf{Q}}_\mathrm{r}^{(t)}$ and $\hat{\mathbf{Q}}_k^{(t)},\ \forall k$
12: Calculate $\epsilon^{(t)}$ from (20)
13: **end while**
14: **return** $\lambda\left(\mathrm{tr}(\hat{\mathbf{Q}}_\mathrm{r}^{(t)}) + \sum_{k=1}^K \mathrm{tr}(\hat{\mathbf{Q}}_k^{(t)})\right)$

V. Numerical Results

In this section we present the numerical results for the multi-pair MIMO relay communication considering Gaussian signaling. We assume $K = 2$ and $N_\mathrm{t} = N_\mathrm{r} = 2$. Here, we assume receiver noise variance at each receive antenna to be equal to 1. The relay transmitter noise variance coefficient is assumed to be $\kappa = 0.1$. Furthermore, we assume that the RSI channel is 10 times stronger than the inter-user interferences. By applying Algorithm 1, we iteratively solve the power minimization problem for the real-valued represented system model. Fig. 2 depicts the minimum required sum-power for given QoS demands per communication pair. Here, we consider the case where all users have equal rate demands for simplicity of illustration. Furthermore, we compare the performance of improper Gaussian signaling with proper Gaussian signaling to emphasize the performance gain. We consider analytical beamforming strategies, namely zero-forcing (ZF) and maximum-ratio transmission/combining (MRT/MRC) for the sake of comparison. It is important to note that, by fixing the beamforming directions to ZF and MRT/MRC, the power minimization problem under rate constraints reduces to a linear program (LP) where the power allocation problem is solved. Fig. 2 approves the fact that MRT/MRC outperforms ZF in noise-limited scenarios (for scenarios with sufficiently low transmit sum-power MRT/MRC achieves the performance of optimal proper Gaussian signaling which is intuitive). However, at sufficiently high transmit power, zero forcing the interference is the optimal beamforming solution (solution from ZF approaches optimal solution for the proper Gaussian signaling at sufficiently high transmit sum-power). Ignoring the transmitter noise at the relay, i.e., $\kappa = 0$, ZF beamforming is the optimal beamforming strategy at relay in case of high transmission power. This can be seen from Fig. 2(a), where both optimal proper and improper Gaussian signaling approach the solution of ZF for high transmission power. Furthermore, improper Gaussian transmission admits a symmetric Gaussian solution (equal power allocation for real and imaginary while uncorrelated), for the rate demands that are optimally achieved by ZF. This is due to the fact that, if sufficient degrees-of-



(a) Transmitter noise variance coefficient $\kappa = 0$

(b) Transmitter noise variance coefficient $\kappa = 0.1$

Fig. 2. Minimum required sum-power for a two-pair MIMO relay network for given rate demands per pair. Here we assumed equal rate demand for the pairs for the sake of compact illustration. The minimum required sum-power in case of improper Gaussian signaling is compared with proper Gaussian signaling. Besides, the performance of proper Gaussian signaling with zero-forcing (ZF) and maximum-ratio transmission/combining (MRT/MRC) is illustrated.

freedom (enough antennas at the relay) are available to zero-force all incident interferences in the network, proper Gaussian signaling is optimal and improper Gaussian signalings reduces to a symmetric Gaussian solution. This can be observed from Fig. 2(a). It is important to note that, allocating unequal power to the real and imaginary components of the complex signal is optimal at low transmit power, while equal power allocation is the solution at high transmit power for SI-free scenario. As can be seen from Fig. 2(b) with the presence of transmitter noise at the relay, i.e., $\kappa = 0.1$, improper Gaussian signaling outperforms proper Gaussian signaling. The performance gap increases as a function of rate demand per pair. This is due to the flexibility for single real streaming in improper Gaussian signaling compared to proper Gaussian signaling that does not allow single real streaming. Single real streaming is the optimal transmission strategy by the sources at high transmit power. This is due to the fact that two real dimensions are reserved for projecting strong transmitter noise onto the null-space of the subspace spanned by the desired signals at the relay input. Hence, by improper Gaussian signaling, all rate demands are always achievable by increasing the transmit power (since single streaming and ZF at the relay renders the network to a noise-limited scenario). Notice that, this situation occurs if the antenna array at the relay is not significantly larger than the number of users. That means, in a massive MIMO relay improper Gaussian signaling does not provide significant improvement in power efficiency compared to proper Gaussian signaling at very high signal-to-interference-plus-noise ratio (SINR). This is due to the availability of sufficient spatial dimensions for null-steering the undesired signal at the relay which is the optimal scheme at very high SINR.

VI. Conclusion

In this paper, we studied the benefits of improper Gaussian signaling in a multi-pair MIMO full-duplex relay network, where the sources and destinations are equipped with single antennas. We formulated the power minimization problem under rate constraints as a semidefinite program utilizing conjugate function which serves as an upper-bound for the concave term in the rate expressions. The problem is solved iteratively in order to tighten the upper-bound at each iteration. Finally, the solutions from improper Gaussian signaling is compared with optimal proper Gaussian signaling, zero-forcing and maximum-ratio transmission/combining. We observed the superiority of improper Gaussian signaling in fulfilling any rate demands which is achieved by single real streaming. Since proper Gaussian signaling is incapable of single real streaming, the network renders to an interference limited scenario where high rate demands can not be fulfilled even with infinite transmission powers.

VII. Appendix

A. Improper Gaussian Random Variable

Definition 1: Let x be a complex zero-mean Gaussian random variable. The second order moments of x are defined as

$$Q_x = \mathbb{E}\{xx^*\} = P_{[r]}^2 + P_{[i]}^2, \tag{21}$$

$$Q_x^{'} = \mathbb{E}\{x^2\} = P_{[r]}^2 - P_{[i]}^2 + 2j\alpha_{[ri]}, \tag{22}$$

respectively, where Q_x and $Q_x^{'}$ are the variance and pseudo-variance of x. Note that, $P_{[r]}$, $P_{[i]}$ and $\alpha_{[ri]}$ are the power of the real component, power of the imaginary component and the correlation between the real and the imaginary components.

Remark 1: A proper Gaussian random variable x, has zero pseudo-variance, i.e., $Q_x^{'} = 0$, which is obtained when $P_{[r]} = P_{[r]}$ and $\alpha_{[ri]} = 0$. Relaxing these assumptions, the Gaussian random variable is improper.

Remark 2: Let $\mathbf{x} = [\Re\{x\} \;\; \Im\{x\}]^T$, where $\Re\{x\}$ and $\Im\{x\}$ are the real and imaginary components of x, respectively. Then, covariance and pseudo-covariance matrices, i.e., $\mathbf{Q}_x = \mathbb{E}\{\mathbf{x}\mathbf{x}^H\}$ and $\mathbf{Q}_x^{'} = \mathbb{E}\{\mathbf{x}\mathbf{x}^T\}$, of the real-valued vector

$\mathbf{x}$ are equal. In other words, the second order moment of $\mathbf{x}$ can be thoroughly characterized by its covariance matrix.

B. Real-valued Channel Model

In order to be consistent with the complex-valued system model, we define $\mathbf{G}_{s_l d_k} = \begin{bmatrix} \Re\{\mathbf{h}_{d_k s_l}\} & \Im\{\mathbf{h}_{d_k s_l}\} \\ -\Im\{\mathbf{h}_{d_k s_l}\} & \Re\{\mathbf{h}_{d_k s_l}\} \end{bmatrix}$, while the other channels can also be defined similarly. Hence, the complex-valued SIMO and MISO channels between the sources and the relay and between the relay and the destinations are represented as real-valued MIMO channels.

C. Upper-bound

Let $f : \mathbb{R}^b \to \mathbb{R}$. Furthermore, consider $\mathbf{Z} \in \mathbb{R}^{a\times a}$ to be a Hermitian positive-semidefinite matrix. Then, the conjugate function of the convex function $f(\mathbf{Z}) = \log|\mathbf{Z}^{-1}|$ is given by

$$\begin{aligned} f^*(\mathbf{W}) &= \sup_{\mathbf{Z}\succeq 0} \quad \text{tr}(\mathbf{ZW}) - f(\mathbf{Z}) \\ &= \log|-\mathbf{W}^{-1}| - \text{tr}(\mathbf{I}_a), \end{aligned} \tag{23}$$

which is parametrized by $\mathbf{W} \in \mathbb{R}^{a\times a}$ and is bounded above if $\mathbf{W} \prec 0$, [27]. Now, having $g(\mathbf{Z}) = -f(\mathbf{Z}) = \log|\mathbf{Z}|$, we can express the following inequality,

$$g(\mathbf{Z}) \leq f^*(\mathbf{W}) - \text{tr}(\mathbf{ZW}), \tag{24}$$

which holds $\forall\ \mathbf{Z} \succeq, \mathbf{W} \prec 0$.
We define $\mathbf{\Gamma} = -\mathbf{W}^{-1} \succeq 0$ and $\mathbf{Z} = \mathbf{AXA^T} + \mathbf{BYB^T} + \mathbf{I}_a$ for $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{a\times b}$ and $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{b\times b}$. Plugging (23) in (24) yields (19).

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