On the DoF of the $3 \times 2$ X-Channel with Mixed CSIT

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Abstract—There is an increasing demand of robust interference management methods which provide strong performance even when the channel state information is imperfect. To investigate this aspect, we study a $3 \times 2$ single-input single-output (SISO) X-channel where each transmitter wants to send an independent message to each receiver. We assume that each receiver is allowed to convey perfect outdated and imperfect instantaneous channel state information to the transmitter (CSIT). For this model, we establish a lower bound on the sum degrees-of-freedom (DoF). We show that, mixed CSIT is useful in the sense that even without the availability of perfect CSIT, it provides a sum DoF that is equal to a similar model with perfect CSIT.

I. INTRODUCTION

The ever increasing demand on higher data rates and increased reliability by both mobile providers and customers requires that modern communication systems are increasingly cognizant of their (channel) environment. The quality of CSI plays a pivotal role to circumvent the impairment caused by the wireless channel. Estimating the perfect CSI at the receiver (CSIR) side can be obtained by well known estimation techniques, for example, by pilot signaling. In contrast to CSIR, the quality of CSIT estimates is mainly governed by the nature of the wireless fading channel (slow vs. fast fading), the feedback bandwidth and the number of channel observations in time and frequency. For example, if channel fluctuations occur very rapidly relative to the time required to provide transmitters with channel observations, current channel estimates are expected to be of rather poor quality or may be available with good quality but with more delays.

In his seminal work [1], Maddah-Ali et al. introduced a multiple-input single-output (MISO) broadcast channel and showed that outdated CSI is useful in the sense that it enlarges the DoF region in comparison to no CSIT. This model is extended to study a variety of networks, namely two-user multiple-input multiple-output (MIMO) broadcast channels [2], X-channels with [3] and without feedback [4], and with secrecy constraints in [5]. The results in all these works are built on assuming that only perfect delayed CSI is conveyed to the transmitter; and, completely ignore the impact of temporal channel correlation. This class of channels where imperfect instantaneous and perfect delayed CSI is conveyed to the transmitter is referred to as mixed CSIT model. In [6], the authors consider a $2 \times 2$ X-channel with mixed CSIT and establish a lower bound on the DoF. In this work, we analyze the sum DoF of an $3 \times 2$ X-channel under mixed CSIT.

Studying this model will provide useful insights to develop a better understanding on establishing the achievability of the general class of $K$-user X-channels.

In this paper, we consider a $3 \times 2$ Gaussian X-channel where each node is equipped with a single antenna and mixed CSIT. The rate at high signal-to-noise ratios (SNR) depends, amongst others, on the CSIT. For the one extreme case of perfect CSIT, the authors of [7] have shown that the optimal (sum) DoF for an $M \times 2$ SISO X-channel is $2M/(M+1)$ achievable through interference alignment (IA). For the other extreme case of only delayed CSIT, results on the DoF are limited to the $2 \times 2$ SISO X-channel. Here, authors of [6], show that the DoF of the $2 \times 2$ Gaussian X-channel with only delayed CSIT is $6/5$, under the restriction that only linear encoding schemes are employed at the transmitters. We introduce $\alpha \in [0,1]$ as the parameter that captures the quality of CSIT estimates. The introduction of the CSIT quality parameter $\alpha$ allows bridging the DoF in the range from no current CSIT ($\alpha = 0$) at all to perfect current CSIT ($\alpha = 1$). In this context, one question that arises is on requirements of the CSIT quality for optimal sum DoF performance.

The main contributions of this paper are summarized as follows. We present an achievability scheme for the $3 \times 2$ X-channel with mixed CSIT. The coding scheme follows by utilizing both resources, i.e., perfect delayed CSIT and imperfect instantaneous CSIT appropriately. The key ingredients of this scheme are built on block-Markov encoding that allows for simultaneous transmission of residual interference encoded as common information along with new private information. We show that, mixed CSIT is useful in the sense that even without availability of perfect CSIT, it provides a sum DoF that is equal to the model of [7] with perfect CSIT.

II. SYSTEM MODEL AND MAIN RESULT

We consider a $3 \times 2$ SISO X-channel, as shown in Fig. 1. There are three transmitters and two receivers. All three transmitters send independent messages to both receivers. The $j$-th transmitter ($j = 1,2,3$) wants to transmit messages $W_{1j} \in W_{1j} = \{1,\ldots,2^{nR_{1j}(P)}\}$ and $W_{2j} \in W_{2j} = \{1,\ldots,2^{nR_{2j}(P)}\}$ to receiver 1 and 2, respectively.

A fast fading channel model is assumed, where each receiver knows the perfect instantaneous CSI along with the past CSI.
The channel output received at the $k$-th receiver at time instant $i$ is given by
\[
y_k[i] = \sum_{j=1}^{M=3} h_{kj}[i]x_j[i] + z_k[i],
\]
where $x_j[i] \in \mathbb{C}$ is the transmitted signal from Tx $j$, $h_{kj}[i] \in \mathbb{C}$ represents the channel coefficient from Tx $j$ to Rx $k$. $z_k[i]$ is unit power complex Gaussian noise $\mathcal{CN}(0,1)$ which is i.i.d. across the Rxs and in time. It is assumed that the channel coefficients are independent realizations of a continuous probability distribution such that the coefficients are mutually independent and change independently across every Tx $j = \{1,2,3\}$ is required to satisfy the average power constraint, that is $\mathbb{E}||x_j[i]||^2 \leq P$. The collection of all transmit signals and channel coefficients at time $i$ are denoted as $x[i] := \{x_j[i]\}_{j=1}^{M=3}$ and $\mathbf{H}[i] \in \mathbb{C}^{2 \times 3}$, respectively.

**Definition 1.** A code for the $3 \times 2$ X-channel under a mixed CSIT setting consists of encoding functions
\[
\{ \phi_{ij} : W_{ij} \times W_{2j} \times \{ H[t] \}_{t=1}^{n} \rightarrow \mathbf{H}[i] \rightarrow X_{j} \}_{j=1}^{n},
\]
for $j = 1, 2, 3$ and decoding functions
\[
\psi_{k1} : Y_{k1}^{n} \times \{ \mathbf{H}[t] \}_{t=1}^{n} \times \{ \mathbf{H}[i] \}_{i=1}^{n} \rightarrow \hat{W}_{k1},
\psi_{k2} : Y_{k2}^{n} \times \{ \mathbf{H}[t] \}_{t=1}^{n} \times \{ \mathbf{H}[i] \}_{i=1}^{n} \rightarrow \hat{W}_{k2},
\psi_{k3} : Y_{k3}^{n} \times \{ \mathbf{H}[t] \}_{t=1}^{n} \times \{ \mathbf{H}[i] \}_{i=1}^{n} \rightarrow \hat{W}_{k3},
\]
for $k = 1, 2$. \{ $\mathbf{H}[t]_{t=1}^{n} \}_{t=1}^{n}$ are perfect delayed channel realizations and $\mathbf{H}[i]$ depicts imperfect current channel estimates.

**Definition 2.** The collection of DoF values \{$d_{kj}$\}_{k=1,j=1}^{k=2,j=3} for the $3 \times 2$ X-channel under an mixed CSIT setting is said to be achievable if the underlying code satisfies
\[
\limsup_{n \to \infty} \Pr\{ W_{kj} \neq \hat{W}_{kj} \} = 0,
\]
and has a pre-log factor of the rate
\[
\lim_{P \to \infty} \liminf_{n \to \infty} \log \frac{|W_{kj}(n,P)|}{n \log P} \geq d_{kj}.
\]
Hereby, the sum DoF is defined as $d_{\Sigma} := \sum_{k=1}^{2} \sum_{j=1}^{3} d_{kj}$.

Entries of $\mathbf{H}[i]$ for all time instances can be described by
\[
h_{kj} = \hat{h}_{kj} + \tilde{h}_{kj}.
\]
The estimation error $\hat{h}_{kj}$ has a complex Gaussian distribution with zero mean and variance/mean square error (MSE) $\mathbb{E}||\hat{h}_{kj}||^2 = \mathbb{E}||h_{kj} - \hat{h}_{kj}||^2 = \sigma^2 < 1$. It is assumed that channel estimates $\hat{h}_{kj}$ and $\tilde{h}_{kj}$ are independent. The variance of $\hat{h}_{kj}$ is $1 - \sigma^2$. The current CSIT quality parameter (cf. [9])
\[
\alpha = \lim_{P \to \infty} \frac{-\log \sigma^2}{\log P}
\]
describes the power order of the MSE at high SNR, i.e. $\sigma^2 \geq P^{-\alpha}$. The range $0 \leq \alpha \leq 1$ suffices to characterize the DoF from delayed CSIT only ($\alpha = 0$) to perfect current CSIT ($\alpha = 1$). Note that (3) implies that the channel estimate of all coefficients at all time is of the same quality. This configuration differs from the alternating CSIT case of [10] as well as from the case when the current CSIT quality varies across Txs.

**Theorem 1.** For the $3 \times 2$ X-channel with mixed CSIT, an achievable sum DoF is given by
\[
d_{\Sigma} \geq \min \left\{ \frac{6}{5} + \frac{21\alpha(4-5\alpha)}{10(15-22\alpha)}, \frac{3}{2} \right\}.
\]

**Proof:** The proof of the achievability scheme is provided in section III.

**Remark 1.** For $\alpha = 1$, the model reduces to $3 \times 2$ X-channel with perfect CSIT that recovers the results in [7] Theorem 2. The optimal sum DoF of the $3 \times 2$ X-channel in case of perfect CSIT corresponds to $3/2$.

**Remark 2.** It is interesting to note that for $\alpha_{th} = 3/7$ in [4], mixed CSIT provides the same sum DoF as in [7]. This result shows that mixed CSIT is useful in the sense that even without availability of perfect CSIT, it provides a sum DoF similar to the perfect CSIT setting.

**III. Achievable Scheme for Imperfect Current CSIT**

Similar to schemes in [6], [11], our proposed scheme for a $3 \times 2$ X-channel deploys a multi-phase transmission strategy. In every phase, information about delayed and current CSIT is used to form precoding scalars that diminish the effect of interfering symbols at both Rxs. As the current CSIT is of imperfect quality, it is not always possible to fully remove interference at a particular phase. The remaining residual interference of a phase is therefore encoded as common information that is transmitted along new private symbols in the consecutive phase. The parallel transmission of both common and private information requires appropriate power-rate adjustments and a proper choice of individual phase durations. The transmission scheme terminates if no more interference is observed at either receivers.
In the following subsections, we will describe phases $s = 1, 2, 3, \ldots, S - 1$ and $S$ individually. Each phase $s$ uses $T_s$ channel uses. Desired information symbols for Rx1 and Rx2 are marked as $a$ and $b$, respectively. Note that every phase transmits distinct information symbols. The notation for information symbols does not explicitly account for that. Symbols $a_{jl}$ and $\bar{a}_{jl}$ denote the $l$-th high and low powered information symbol in the considered phase that is intended for Rx1 by the $j$-th Tx. For the sake of brevity, the scheme is described from Rx1’s perspective without explicitly incorporating noise in all preceding expressions. For notational simplicity, we define $T_{\Sigma} := \sum_{s=1}^{S} T_s$ and $T_{\Sigma}^2 := t + T_{\Sigma}^2$ for $3 \leq \pi \leq S$ and $1 \leq t < T_{\Sigma}$. A received signal $y_k(\cdot)$ is the received signal at Rx $k$ after the common information has been removed.

A. Phase 1

Similar to the perfect IA scheme of \cite{11}, phase 1 (cf. x(1 : 4)) uses $M + 1 = 4$ slots to transmit 3 high power information symbols per Rx of order $O(P)$. The remaining symbols are low power information symbols of order $O(P^{1-\alpha})$.

$$x[1] = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} + \bar{a}_{31} \end{bmatrix}, \quad x[2] = \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} + \bar{b}_{31} \end{bmatrix},$$

$$x[3] = \begin{bmatrix} a_{11} + b_{11} \\ \alpha_2^{[1]} a_{21} + \nu_2^{[1]} b_{21} \\ \lambda_3^{[1]} (a_{31} + \bar{a}_{31}) + \nu_3^{[1]} (b_{31} + \bar{b}_{31}) \end{bmatrix},$$

$$x[4] = \begin{bmatrix} a_{11} + b_{11} \\ \alpha_2^{[2]} a_{21} + \nu_2^{[2]} b_{21} \\ \lambda_3^{[2]} (a_{31} + \bar{a}_{31}) + \nu_3^{[2]} (b_{31} + \bar{b}_{31}) \end{bmatrix}.$$ 

Ignoring the low power symbols for $\alpha = 1$, perfect IA is achieved by fixing two beamforming vectors (of dimension $(M + 1) \times 1$) and solving for the remaining four beamforming vectors (cf. (6) in \cite{11}). By setting the beamforming vectors of information symbol $a_{11}$ and $\bar{a}_{11}$ to $[1, 0, 1, 1]^T$ and $[0, 1, 1, 1]^T$, respectively, and all 8 unit power precoding coefficients to

$$\lambda_2^{[1]} = \frac{h_{12}[1] h_{21}[3]}{h_{21}[1] h_{22}[3]}, \quad \lambda_3^{[1]} = \frac{h_{12}[1] h_{21}[3]}{h_{22}[1] h_{23}[3]}, \quad \lambda_3^{[2]} = \frac{h_{12}[2] h_{21}[4]}{h_{21}[2] h_{22}[4]}, \quad \lambda_3^{[3]} = \frac{h_{12}[2] h_{21}[4]}{h_{11}[2] h_{13}[4]},$$

$$\nu_2^{[1]} = \frac{h_{12}[2] h_{11}[3]}{h_{11}[2] h_{12}[3]}, \quad \nu_2^{[2]} = \frac{h_{12}[2] h_{11}[3]}{h_{11}[2] h_{12}[3]}, \quad \nu_2^{[3]} = \frac{h_{12}[2] h_{11}[3]}{h_{11}[2] h_{12}[3]}, \quad \nu_2^{[4]} = \frac{h_{12}[2] h_{11}[3]}{h_{11}[2] h_{12}[3]},$$

perfect IA becomes feasible (for $\alpha \geq 1$). Interference alignment allows Rx $k = \{1, 2\}$ to do the following step to be free of interference ($t_1 = 2, t_2 = 1$)

$$\Delta_k(3; t_k) := y_k[3]/h_{k1}[3] - y_k[t_k]/h_{k1}[t_k],$$

$$\Delta_k(4; t_k) := y_k[4]/h_{k1}[4] - y_k[t_k]/h_{k1}[t_k].$$

The received signals (ignoring noise) for Rx1 take the form

$$y_1[1] = h_{11}[1] a_{11} + h_{12}[1] a_{21} + h_{13}[1] (a_{31} + \bar{a}_{31}),$$

$$y_1[2] = h_{11}[2] b_{11} + h_{12}[2] b_{21} + h_{13}[2] (b_{31} + \bar{b}_{31}),$$

$$y_1[3] = h_{11}[3] (a_{11} + b_{11}) + h_{12}[3] (\lambda_2^{[1]} a_{21} + \nu_2^{[1]} b_{21}) + \lambda_3^{[1]} (a_{31} + \bar{a}_{31}) + \nu_3^{[1]} (b_{31} + \bar{b}_{31}),$$

$$y_1[4] = h_{11}[4] (b_{11} + b_{21}) + h_{12}[4] (\lambda_2^{[2]} a_{21} + \nu_2^{[2]} b_{21}) + \lambda_3^{[2]} (a_{31} + \bar{a}_{31}) + \nu_3^{[2]} (b_{31} + \bar{b}_{31}).$$

For Rx1, the second equation of (10) is pure interference that is used in (8) and (9) to cancel all interference components generated by Tx1 at channel use 3 and 4. From Rx1’s perspective, the differences in (8) and (9) are given by (5) and (6) on the top of the next page, respectively. Rx1 observes two interference components from Txs 2 and 3. These linear equations represent interference that is useful for Rx1 and Rx2. On the one hand, the interference components are helpful for Rx1 to remove residual interference in $\Delta_1(3; 2)$ and $\Delta_1(4; 2)$. On the other hand, these equations are functions that depend solely on symbols dedicated to Rx2. Similarly, Rx2 generates four additional interference components by performing (8) and (9). Hence, there are 8 interference components in total.

A naive way of delivering all interference components for interference cancelation is to provide them in an analog fashion. In [9], [12], it is shown that quantizing every interference component while keeping the quantization noise at noise level is sufficient for error-free decoding. Since the interference components $I_{12}^{[1]}(b_{21})$ and $I_{12}^{[2]}(b_{21})$ (analogously from Rx2’s perspective), that are generated in the first 4 transmission slots of phase 1, are redundant, only 6 instead of 8 interference components (including 2 components by Tx2 and 4 components by Tx3) have to be provided to Rx2 and Rx3 from Tx2 and Tx3 at a ratio of 1 : 2. The ratio can be modified to 1 : 1 if the first four slots are repeated with changes in roles of Tx2 and Tx3. If the first phase uses $T_1$ slots ($T_1 \geq 8$) in total, one can repeat the first 8 slots $T_1/8$ times with new full and low power information symbols. The total number of quantization bits generated in phase 1 are therefore $16T_1(1 - \alpha) \log P$ bits. Every interference component in (5) and (6) requires $(1 - \alpha) \log P$ quantization bits. For sake of brevity, we show this fact for $I_{12}^{[1]}(b_{21})$ by considering the power order of its effective coefficient using (3) and (7)

$$E \left\| \frac{h_{12}[3]}{h_{11}[3]} \nu_2^{[1]} - \frac{h_{12}[2]}{h_{11}[2]} \nu_2^{[2]} \right\|_2^2 =$$

$$E \left\| \frac{h_{12}[2]}{h_{11}[2]} \right\|_2^2 E \left\| \frac{h_{12}[3] h_{11}[3]}{h_{11}[3] h_{12}[3]} - 1 \right\|_2^2 =$$

$$E \left\| \frac{h_{12}[3] h_{11}[3]}{h_{11}[3] h_{12}[3]} \right\|_2^2 \leq P^{1-\alpha}.$$
\[ \Delta_1(3;2) = a_{11} + \frac{h_{12}[3]}{h_{11}[3]} \lambda_1^{[1]}(a_{21} + \tilde{a}_{32}) + \left( \frac{h_{12}[3]}{h_{11}[3]} T_{21} - \frac{h_{12}[2]}{h_{11}[2]} \right) b_{21} + \frac{h_{13}[3]}{h_{11}[3]} \rho_{31}[1] b_{32} - \frac{h_{13}[2]}{h_{11}[2]} b_{31} \] 

\[ \Delta_1(4;2) = a_{11} + \frac{h_{12}[4]}{h_{11}[4]} \lambda_1^{[2]}(a_{21} + \tilde{a}_{32}) + \left( \frac{h_{12}[4]}{h_{11}[4]} T_{21} - \frac{h_{12}[2]}{h_{11}[2]} \right) b_{21} + \frac{h_{13}[4]}{h_{11}[4]} \rho_{31}[2] b_{33} - \frac{h_{13}[2]}{h_{11}[2]} b_{31} \]
and for slot $T_1 + 6$ equals to

$$\chi_{21}^{[3]} = \frac{h_{21}[T_1 + 6]}{h_{21}[T_1 + 6]} \left(\frac{h_{22}[T_1 + 3]h_{21}[T_1 + 3]}{h_{21}[T_1 + 3]h_{22}[T_1 + 3]} - 1\right) \frac{h_{22}[T_1 + 1]}{h_{21}[T_1 + 1]}.$$  

$$\chi_{21}^{[3]} = \frac{h_{21}[T_1 + 6]}{h_{21}[T_1 + 6]} \left(\frac{h_{22}[T_1 + 3]h_{21}[T_1 + 3]}{h_{21}[T_1 + 3]h_{22}[T_1 + 3]} - 1\right) \frac{h_{22}[T_1 + 1]}{h_{21}[T_1 + 1]}.$$  

$$\chi_{22}^{[2]} = \frac{h_{22}[T_1 + 3]h_{21}[T_1 + 6]}{h_{21}[T_1 + 3]h_{22}[T_1 + 6]} \frac{h_{22}[T_1 + 1]}{h_{21}[T_1 + 1]}.$$  

$$\nu_{21}^{[3]} = \frac{h_{31}[T_1 + 6]}{h_{21}[T_1 + 6]} \left(\frac{h_{12}[T_1 + 3]h_{11}[T_1 + 3]}{h_{11}[T_1 + 3]h_{12}[T_1 + 3]} - 1\right) \frac{h_{13}[T_1 + 2]}{h_{11}[T_1 + 2]}.$$  

$$\nu_{21}^{[3]} = \frac{h_{31}[T_1 + 6]}{h_{21}[T_1 + 6]} \left(\frac{h_{12}[T_1 + 3]h_{11}[T_1 + 3]}{h_{11}[T_1 + 3]h_{12}[T_1 + 3]} - 1\right) \frac{h_{13}[T_1 + 2]}{h_{11}[T_1 + 2]}.$$  

$$\nu_{22}^{[2]} = \frac{h_{12}[T_1 + 3]h_{11}[T_1 + 6]}{h_{11}[T_1 + 3]h_{12}[T_1 + 6]} \frac{h_{13}[T_1 + 3]h_{11}[T_1 + 6]}{h_{11}[T_1 + 3]h_{13}[T_1 + 6]}.$$  

The interference observed in (23)–(25) reduces to noise level $O(P^0)$. Some precoding coefficients (e.g. $\lambda_{11}^{[1]}$) in (16)–(18) have power order $O(P^{-\alpha})$. In the following, the power reduction factor for one interference term caused by Tx2 in (23) is considered ($\chi := \frac{\nu_{21}^{[3]}h_{12}[T_1 + 3]}{\nu_{21}^{[3]}h_{13}[T_1 + 3]} - \frac{h_{12}[T_1 + 2]}{h_{13}[T_1 + 2]}$):

$$E\left[\left|\begin{array}{c}
\chi - h_{12}[T_1 + 4]h_{13}[T_1 + 4] \nu_{21}^{[2]} \\
\frac{h_{12}[T_1 + 4]}{h_{13}[T_1 + 4]} \chi
\end{array}\right|^2\right] = E\left[\left|\begin{array}{c}
\chi - h_{12}[T_1 + 4]h_{13}[T_1 + 4] \nu_{21}^{[2]} \\
\frac{h_{12}[T_1 + 4]}{h_{13}[T_1 + 4]} \chi
\end{array}\right|^2\right] = E\left[\|\chi\|^2\right] E\left[\left|\begin{array}{c}
1 - \frac{h_{12}[T_1 + 4]}{h_{13}[T_1 + 4]} \frac{\nu_{21}^{[2]}h_{12}[T_1 + 4]}{h_{13}[T_1 + 4]}
\end{array}\right|^2\right] \approx P^{-2\alpha}.$$  

The last step in (19) follows from (11) and $E\left[\|\chi\|^2\right] \approx P^{-\alpha}$. Repeating the first 12 slots of the second phase $T_2/12$ times, will therefore construct $T_2/8\sigma\log P$ quantization bits that are transmitted as common information by Tx2 and Tx3 at a ratio of 1 : 1 in phase 3. The number of slots required for phase 2 is determined by the fact that the total number of quantization bits fabricated in phase 1 have to be "consumed" in $T_2$ slots, i.e. the ratio $\frac{12}{T_1}(1 - \alpha)\log P / \frac{12}{T_2}(2 - 3\alpha)\log P$ has to be 1. This is equivalent to

$$T_2 = \frac{3(1 - \alpha)}{2 - 3\alpha} T_1.$$  

2) Phases $s = 3, 4, \ldots, S - 1$: In phase 2, the ratio of common information to be transmitted from Tx2 and Tx3 is 1 : 1. This ratio arises from the interference (originating from Tx2 and Tx3) seen at both Rxs in the previous phase. Phase 2, itself, also creates interference at Tx2 and Tx3 at a ratio of 1 : 1. This implies that in phases 3, 4, \ldots, S - 1, the same transmission strategy as in phase 2 can be simply repeated. The time relationship between phase 2 and 3 for example becomes

$$T_3 = \frac{4\alpha}{3(2 - 3\alpha)} T_2.$$  

In order to compute the DoF, it is required to compute an expression for $T_2^S$ as a function of $\sigma$ and $\mu$.

$$T_2^S = T_1 + T_2 + T_3 + \ldots + T_{S-1} = T_1 + T_2 + T_2 \cdot \sum_{i=1}^{S-3} \mu_i,$$  

(27)

where for large $S$ and $0 \leq \mu \leq 1$, one can easily show that

$$T_2^S = T_1 \left(1 + \frac{\sigma}{\mu - 1}\right).$$  

(28)

C. Phase S

The residual interference created in phase $S - 1$ is compressed to common information by Tx2 and 3 at a ratio of 1 : 1. Therefore, it is sufficient to use 4 slots and 3 information symbols per Rx. The channel outputs at Rx1 for this case ($m = 3, 4$) is given in (29).

$$x[T_2^{S,1}] = \begin{bmatrix} a_{11} \\ a_{21} \\ c_1 + a_{31} \end{bmatrix}, \quad x[T_2^{S,2}] = \begin{bmatrix} b_{11} \\ b_{21} \\ c_2 + b_{31} \end{bmatrix},$$

$$x[T_2^{S,3}] = \begin{bmatrix} a_{11} + b_{11} \\ c_3 + \lambda_{21}^{[1]} a_{21} + \nu_{21}^{[1]} b_{21} \\ \lambda_{31}^{[2]} a_{31} + \nu_{31}^{[2]} b_{31} \end{bmatrix},$$

$$x[T_2^{S,4}] = \begin{bmatrix} a_{11} + b_{11} \\ c_4 + \lambda_{21}^{[2]} a_{21} + \nu_{21}^{[2]} b_{21} \\ \lambda_{31}^{[2]} a_{31} + \nu_{31}^{[2]} b_{31} \end{bmatrix}.$$  

(29)

All precoding coefficients in phase $S$ are chosen in the exact same way as in phase 1 (cf. (7)). When computing ($t_1 = T_2^{S,2}, t_2 = T_2^{S,1}$) $\Delta_k[T_2^{S,m}; t_k] = \frac{y_k}{h_{31}^{[1]} T_2^{S,m}} - \frac{y_k}{h_{31}^{[1]} t_k}$, the power of residual interference terms are at most at noise power level (cf. (31)). The normalized sum rate of common information equals $4(1 - \alpha)$. Then, the relationship between $T_{S-1}$ and $T_S$ becomes

$$T_3 = \frac{2\alpha}{3(1 - \alpha)} T_{S-1}.$$  

(30)
\[
\Delta_1(T_1 + 3; T_1 + 5, T_1 + 2) = \left( \frac{h_{11}^2[T_1 + 3]}{\nu_{11}^2 h_{12}[T_1 + 3]} - \frac{h_{11}^2[T_1 + 5]}{h_{22}[T_1 + 3]} \lambda_{11}^a \right) a_{11} + \frac{\lambda_{31}^a h_{13}[T_1 + 3]}{\nu_{11}^2 h_{12}[T_1 + 3]} b_{31} + \left( \frac{1}{\nu_{11}^2} - \lambda_{21}^a \right) a_{21} +
\]

\[
D_{13}^a(a_{11}, a_{21}, a_{31}, \bar{a}_{22}, \bar{a}_{32})
\]

\[
\Delta_1(T_1 + 3; T_1 + 6, T_1 + 2) = a_{11} + \left( \frac{h_{12}^2[T_1 + 3]}{h_{11}[T_1 + 3]} \lambda_{32}^a \right) a_{31} + \frac{h_{13}[T_1 + 3]}{h_{12}[T_1 + 3]} \lambda_{32}^a a_{21} + \left( \frac{h_{12}^2[T_1 + 3]}{h_{11}[T_1 + 3]} \lambda_{32}^a \right) a_{31} + \left( \frac{h_{13}[T_1 + 3]}{h_{12}[T_1 + 3]} \lambda_{32}^a \right) a_{31} + \left( \frac{h_{13}[T_1 + 3]}{h_{12}[T_1 + 3]} \lambda_{32}^a \right) a_{31} + \left( \frac{h_{13}[T_1 + 3]}{h_{12}[T_1 + 3]} \lambda_{32}^a \right) a_{31} +
\]

\[
D_{13}^a(a_{11}, a_{21}, a_{31}, \bar{a}_{22}, \bar{a}_{32})
\]

\[
\Delta_1(T_{S}^v, T_{S}^w) = a_{11} + \frac{h_{12}^2[T_{S}^v]}{h_{11}[T_{S}^v]} \lambda_{32}^a a_{21} +
\]

\[
\frac{h_{13}[T_{S}^m]}{h_{11}[T_{S}^m]} \lambda_{32}^a a_{31} + \frac{h_{12}^2[T_{S}^m]}{h_{11}[T_{S}^m]} \lambda_{32}^a b_{21} + \left( \frac{h_{13}[T_{S}^m]}{h_{11}[T_{S}^m]} \lambda_{32}^a \right) a_{31} +
\]

\[
D_{13}^a(a_{11}, a_{21}, a_{31}, \bar{a}_{22}, \bar{a}_{32})
\]

\[
\Delta_1(T_{S}^v, T_{S}^w) = a_{11} + \frac{h_{12}^2[T_{S}^v]}{h_{11}[T_{S}^v]} \lambda_{32}^a a_{21} +
\]

\[
\frac{h_{13}[T_{S}^m]}{h_{11}[T_{S}^m]} \lambda_{32}^a a_{31} + \frac{h_{12}^2[T_{S}^m]}{h_{11}[T_{S}^m]} \lambda_{32}^a b_{21} + \left( \frac{h_{13}[T_{S}^m]}{h_{11}[T_{S}^m]} \lambda_{32}^a \right) a_{31} +
\]

\[
D_{13}^a(a_{11}, a_{21}, a_{31}, \bar{a}_{22}, \bar{a}_{32})
\]

\[
D_{13}^a(a_{11}, a_{21}, a_{31}, \bar{a}_{22}, \bar{a}_{32})
\]

D. Backwards Decoding

Note that common information that is required for successful decoding at phase 1 is transmitted at its consecutive phase, i.e. phase 2 + 1. For that reason, the scheme deploys a backwards decoding scheme. In the following, the (backwards) decoding procedure is described in further detail from the perspective of Rx1.

1) Phase S: In this phase, TIN is applied beforehand to decode the common information that is needed to discard the observed interference in phase S – 1. The common information can then be removed from the received signals. As this phase does not generate extra interference terms, Rx1 (similarly for Rx2) experiences a noise-bounded 3 x 3 MIMO channel with equivalent matrix \( A_S \). The matrix is full rank almost surely. This leads to a proper decoding of \( a_{11}, a_{21} \) and \( a_{31} \).

2) Phase 2: Again, Rx1 applies TIN beforehand to decode the common information that is needed to discard the observed interference in phase 1. The common information can then be removed from the received signals. Interference from Rx1’s perspective that is encoded as common information in phase 3 is available to “clean” the received signals that are interference-distorted (cf. row 2 of \( A_2 \) with \( \bar{a}_{22} \)). Interference that Rx2 experiences in the second phase is useful information for Rx1 (cf. row 3–4 of \( A_2 \)). The last three rows of \( A_2 \) are the direct result of

\[
D_{13}^a(\cdot) - \frac{h_{13}[T_{S}^v]}{h_{12}[T_{S}^v]} D_{13}^a(\cdot),
\]

\[
D_{13}^a(\cdot) - \frac{h_{13}[T_{S}^v]}{h_{12}[T_{S}^v]} D_{13}^a(\cdot),
\]

\[
D_{13}^a(\cdot) - D_{13}^a(\cdot),
\]
resulting full rank matrix $A$ in phase 2 resembles an $7 \times 7$ MIMO channel. The resulting full rank matrix $A_2$ allows for successful decoding of $a_{11}, a_{21}, \ldots, a_{32}$ in 6 slots.

3) Phase 1: In this phase, TIN is not required as it is the first phase that does not provide any common information. The total number of interference components in 4 slots is kept at 6 (i.e. 3 per Rx). This generates an $6 \times 6$ MIMO channel (cf. $A_1$) allowing for error-free decoding of information symbols $a_{11}, a_{21}, \ldots, a_{33}$ in 4 slots.

E. Resulting DoF

It is straightforward to compute the normalized sum rate of private information for all $S$ phases that is achieved in $\sum_{i=1}^{S} T_i = T_T^S + T_S$ slots. It is given by:

$$
\frac{T_i}{8} (24 - 12\alpha) + \sum_{i=2}^{S-1} \frac{T_i}{12} 40\alpha + \frac{T_S}{4} 6\alpha.
$$

The resulting (sum) DoF becomes

$$
d_s = \frac{10}{3} \alpha + \frac{16\alpha}{1 + \sigma \frac{1}{1-\mu} + \mu^{3\gamma} (\gamma - \frac{\mu}{1-\mu})}.
$$

For large $S$ and $0 \leq \mu \leq 1$, the DoF expression as a function of $\alpha$ corresponds to

$$
d_s = \min \left\{ \frac{6}{5} + \frac{21\alpha (4 - 5\alpha)}{10(15 - 22\alpha)}, \frac{3}{2} \right\}.
$$

It suffices that the CCSIT quality parameter is $\alpha_{CSIT} = \frac{3}{4}$ to achieve the optimal sum DoF of $\frac{3}{2}$. In comparison to the results in [6] for the $2 \times 2$ case, the requirement on the CCSIT quality parameter reduces by about 3.5%.

IV. CONCLUSION

In this work, we study a $3 \times 2$ X-channel with mixed CSIT. We establish a lower bound on allowed sum DoF. We show that even without availability of perfect CSIT, mixed CSIT provides a sum DoF similar to perfect CSIT setting. This result highlights the fact that, in certain situations, perfect CSIT may not provide larger sum DoF in comparison to mixed CSIT.
settings. The results that we have establish in this work will provide new insights to study the general class of $K$-user X-channels.

REFERENCES


