Interference MAC: Impact of Improper Gaussian Signaling on the Rate Region Pareto Boundary

Ali Kariminezhad, Anas Chaaban, and Aydin Sezgin

Abstract—**Meeting the challenges of 5G demands better exploitation of the available spectrum by allowing multiple parties to share resources. For instance, a secondary unlicensed system can share resources with the cellular uplink of a primary licensed system for an improved spectral efficiency. This induces interference which has to be taken into account when designing such a system. A simple yet robust strategy is treating interference as noise (TIN), which is widely adapted in practice. It is thus important to study the capabilities and limitations of TIN in such scenarios. In this paper, we study this scenario modelled as multiple access channel (MAC) interfered by a Point-to-Point (P2P) channel, where we focus on the characterization of the rate region. We use improper Gaussian signaling (instead of proper) at the transmitters to increase the design flexibility, which offers the freedom of optimizing the transmit signal pseudo-variance in addition to its variance. We formulate the weighted max-min problem as a semidefinite program, and use semidefinite relaxation (SDR) to obtain a near-optimal solution. Numerical optimizations show that, by improper Gaussian signaling the achievable rates can be improved upto three times when compared to proper Gaussian signaling.**

Index Terms—**Improper Gaussian signaling, rate maximization, partial interference multiple access channel, Pareto boundary, augmented covariance matrix.**

I. Introduction

The continuous increase in the demand for high data rates is a challenging issue that confronts today's communication systems. This challenge needs to be addressed in order to enable future systems to cope with this increasing demand. One way to tackle this problem is by allowing resource sharing, where multiple users/systems share the same spectrum in order to achieve better performance. By allowing this paradigm of resource sharing, networks naturally become more heterogeneous and more interference-limited. Nevertheless, by allowing shared resources, the performance can be better in comparison to isolating systems by allocating orthogonal resources. This follows since the negative impact of interference can be overcome by the positive impact of increased bandwidth if the transmission is designed properly. As an example, we can think of a primary cellular network sharing resources with another system such as a Device-to-Device (D2D) communication system, a small cell [1], or more generally a secondary cognitive radio [2]. Fig. 1 depicts a scenario with both D2D nodes and a small cell sharing resources with a cellular network operating in uplink phase.

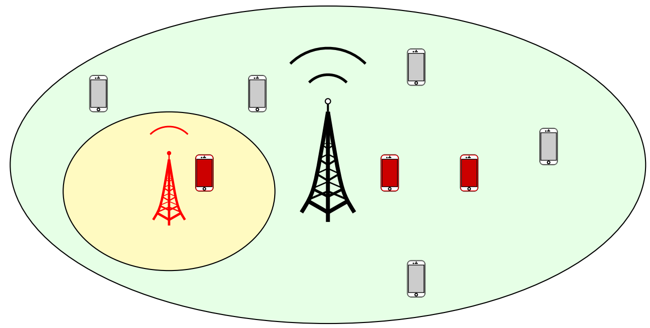

Fig. 1: Multiple access channel interfered by communication in a small cell (in yellow) or by a Device-to-Device communication (in red).

In this paper, we focus on this aspect in a cellular uplink with shared resources. Namely, we study a network consisting of a MAC sharing its resources with a P2P channel referred to as the partial interfering multiple access channel (PIMAC) as depicted in Fig. 2, [3]–[5]. Partial interference MAC is insightful for more sophisticated networks such as X-channel. Furthermore, it models the simplest heterogeneous network with two tiers, namely a MAC and a D2D communication. As stated earlier, the P2P channel can represent an underlay cognitive system, a pair of D2D communicating devices, or a small cell. Here, the P2P channel is active only if it does not deteriorate the QoS of the primary MAC users [6].

To guarantee good performance, the receivers can employ different interference management strategies. The receivers can either decode interference and subtract it from the received signal to extract the intended signal [5], [7], or simply treat this interference as noise, (TIN) [8]. Interestingly, TIN was shown to be optimal for the two-user interference channel (IC) under certain conditions [9], [10]. Optimality conditions of TIN in the PIMAC were investigated in [11] where the constant-gap optimality of TIN is studied. We focus on TIN due to its practical simplicity, robustness, and good performance in many practical scenarios.

In this work, we consider a generalized version of TIN which incorporates improper Gaussian signaling [12], [13] instead of the classical proper Gaussian signaling. Compared to proper signaling, improper signaling enables improving the achievable rates of the PIMAC since it enjoys the additional freedom of designing the pseudo-variance in addition to the variance of the transmit signal. For instance, improper signaling was proposed in [14] as a means to improve the degrees

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of freedom (DoF) in the 3-user IC. In comparison, for the 2-user IC improper signaling does not enhance the DoF, yet improper signaling is useful in the low and moderate SNR (signal to noise power ratio) regime, as it improves the signal-to-interference-plus-noise ratio (SINR) as shown in [15], [16]. In these papers, the authors show that the rate region of the 2-user IC is improved by Gaussian improper signaling compared to Gaussian proper signaling. Moreover, the authors in [17] study the benefits of improper Gaussian signaling in two-tier full-duplex network from the rate-energy perspective. Furthermore, the authors in [18] investigate the benefits of improper Gaussian signaling in a two-hop alternate relaying system without having the channel state information at the source.

The PIMAC considered here can be seen as a generalization of the elemental 2-user IC. For instance, using time division multiple access (TDMA) for the MAC users, the PIMAC can be viewed as a set of separate ICs. Instead of TDMA, in this paper we focus on the general scenario where the users are allowed to simultaneously share the spectrum. The achievable rate tuples of the network are to be determined under this consideration. To this end, we utilize the so-called rate-profile method proposed in [19] to characterize the Pareto boundary of the achievable rate region. Herein, the Pareto boundary defines the frontier of the achievable rate region, where an increment in the rate of one user inevitably coincides with a decrement in the rate of at least one of the other users. The problem of characterizing the Pareto boundary by the rate-profile method is non-convex. To overcome this problem, we reformulate the optimization problem as a semidefinite program (SDP) with rank constraints. The reformulated problem is non-convex due to the rank constraints which are then relaxed. The semidefinite relaxation (SDR) is then solved efficiently by interior point methods (i.e., barrier methods [20]). Note that, the optimal solution of SDR may not satisfy the rank constraints of the original problem and we need to determine an approximate solution by the so-called Gaussian randomization process [16]. By numerical evaluation, we demonstrate that under both weak and strong interference, improper signaling improves the Pareto boundary compared to proper signaling. This improvement becomes more apparent when the interference gets stronger.

A. Notation

Throughout the paper, we represent vectors in boldface lower-case letters while the matrices are expressed in boldface upper-case. $\mathrm{Tr}(\mathbf{A})$, $|\mathbf{A}|$, $\mathbf{A}^H$, $\mathbf{A}^*$, $\mathbf{A}^T$ represent the trace, determinant, hermitian, complex conjugate and transpose of matrix $\mathbf{A}$, respectively.

II. System Model

The system under investigation, consists of a cellular system operating in the uplink which shares spectrum with a P2P channel. This is modeled as a 3-user PIMAC, consisting of a MAC with two users and a P2P channel. The input-output relation at any given transmission instant can be written as

$$y_1 = \underbrace{\sum_{j=1}^{2} h_{1j}x_j}_{\text{desired signal}} + \underbrace{h_{13}x_3 + z_1}_{\text{interference + noise}=s_1}, \quad (1)$$

$$y_2 = \underbrace{h_{23}x_3}_{\text{desired signal}} + \underbrace{\sum_{j=1}^{2} h_{2j}x_j + z_2}_{\text{interference + noise}=s_2}, \quad (2)$$

where h_{ij} denotes the complex-valued channel from the j^{th} transmitter to the i^{th} receiver, z_i represents zero-mean additive white Gaussian noise with variance σ^2, i.e., $z_i \sim \mathcal{CN}(0, \sigma^2)$, $x_j \in \mathbb{C}$ stands for the complex transmit signal from the j^{th} transmitter and y_i is the received signal at the i^{th} receiver. The transmit signals satisfy a power constraint $\mathbb{E}[|X_i|^2] \leq P_i$. We assume that transmitter i encodes an independent message of rate R_i, and transmits it over the shared medium. The MAC users communicate with their receiver (a base station (BS)), which receives interference from the P2P channel transmitter, and similarly, the P2P communication observes interference from the MAC users. Note that, the interference-plus-noise terms at the first and second receivers are denoted by s_1 and s_2, respectively.

III. Rate Maximization

Assuming that the receivers treat interference as noise (TIN), and that the MAC receiver uses a MAC-optimal decoding strategy (such as successive decoding combined with time-sharing), we can express the achievable rates of the MAC users as the set of (R_1, R_2) bounded by [21]

$$R_1 \leq I(X_1; Y_1|X_2), \quad (3)$$

$$R_2 \leq I(X_2; Y_1|X_1), \quad (4)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_1), \quad (5)$$

where $I(X_i; Y_1|X_j)$ is the mutual information between X_i and Y_1 given X_j, and $I(X_1, X_2; Y_1)$ is the mutual information between (X_1, X_2) and Y_1. The third user (P2P user) achieves the following rate by TIN

$$R_3 \leq I(X_3; Y_2). \quad (6)$$

Assume that all users in the PIMAC generate their transmit signals from a Gaussian codebook. A Gaussian random variable (RV) is completely characterized by the first-order and second-order moments. The variance of X can be expressed as $C_X = \mathbb{E}[|X|^2]$, and it completely characterizes the second-order moment of the complex Gaussian RV if and only if it is proper [12]. The main idea of improper signaling is to allow non-equal power allocation over the real and imaginary components of the transmit signals and allow them to be correlated. The variance in this case does not characterize the second-order moment thoroughly since it does not capture the real-imaginary correlation between the real and imaginary components. Instead, the second-order moment is described by the augmented covariance matrix defined next.

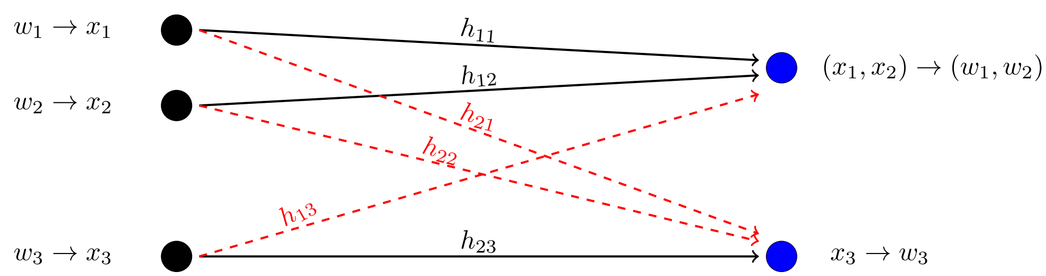


Fig. 2: A Partial Interference Multiple Access Channel (PIMAC). The transmit signals x_j are a function of the messages w_j which are the realizations from the Gaussian codebook.

Definition 1 ([12]). *The second-order moment of an improper Gaussian RV X is described by the augmented covariance matrix*

$$\hat{\mathbf{C}}_X = \begin{bmatrix} C_X & \tilde{C}_X \\ \tilde{C}_X^* & C_X \end{bmatrix}, \tag{7}$$

where, $\tilde{C}_X = \mathbb{E}[X^2]$ is the pseudo-variance of X.

Furthermore, the improper Gaussian random variable, has the following entropy,

Definition 2 ([12]). *The entropy of an improper Gaussian RV X is*

$$h(X) = \frac{1}{2}\log((2\pi e)^2 |\hat{\mathbf{C}}_X|). \tag{8}$$

The mutual information terms mentioned above can be recast as the subtraction of two entropy terms. With (8), we can state the following for the P2P user,

$$\begin{aligned} R_3 \leq I(X_3; Y_2) &= h(Y_2) - h(Y_2|X_3) \\ &= \frac{1}{2}\log\frac{|\hat{\mathbf{C}}_{y_2}|}{|\hat{\mathbf{C}}_{s_2}|} \\ &= \frac{1}{2}\log\frac{C_{y_2}^2 - |\tilde{C}_{y_2}|^2}{C_{s_2}^2 - |\tilde{C}_{s_2}|^2} = L_3, \end{aligned} \tag{9}$$

where s_2 is defined in (2). Note that this rate is achievable by improper Gaussian signaling at the transmitter and TIN at the receiver. For the MAC users, the achievable rates using improper signaling can be written similarly as

$$R_1 \leq \frac{1}{2}\log\frac{C_{y_{12}}^2 - |\tilde{C}_{y_{12}}|^2}{C_{s_1}^2 - |\tilde{C}_{s_1}|^2} = L_1, \tag{10}$$

$$R_2 \leq \frac{1}{2}\log\frac{C_{y_{11}}^2 - |\tilde{C}_{y_{11}}|^2}{C_{s_1}^2 - |\tilde{C}_{s_1}|^2} = L_2, \tag{11}$$

$$R_1 + R_2 \leq \frac{1}{2}\log\frac{C_{y_1}^2 - |\tilde{C}_{y_1}|^2}{C_{s_1}^2 - |\tilde{C}_{s_1}|^2} = L_4, \tag{12}$$

where, $y_{ij} = y_i - h_{ij}x_j$ and C_{s_1},$\hat{C}_{s_1}$, C_{s_2}, $\hat{C}_{s_2}$ are the variance and pseudo-variance of the expressions in (1), (2). The variables L_1, L_2, L_3 and L_4 are defined for future use in the upcoming optimization problems.

Note that if the P2P user is silent, the system reduces to a Gaussian MAC channel, for which the capacity region can be achieved by proper Gaussian signaling [21]. If the P2P user is active however, the achievable rate region of the MAC shrinks. It is interesting to quantify this trade-off between R_3 and the set of achievable rates (R_1, R_2). This can be done by studying the Pareto boundary of the achievable rate region.

To characterize the Pareto boundary of the rate region, consider a sum rate $R_\Sigma(\boldsymbol{\alpha})$ with $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \alpha_3] \in [0, 1]^3$ such that $\sum_{j=1}^{3} \alpha_j = 1$, so that the users' achievable rates can be expressed as

$$R_j = \alpha_j R_\Sigma(\boldsymbol{\alpha}). \tag{13}$$

The vector $\boldsymbol{\alpha}$ is called the target rate-profile vector. By scanning through feasible rate-profile vectors and maximizing $R_\Sigma(\boldsymbol{\alpha})$, we acquire the complete Pareto boundary of the rate region [19].

Having defined all the necessary quantities, we can formulate the sum rate maximization problem in a particular scanning direction (i.e., target rate-profile vector) as follows:

$$\max_{C_{x_j}, \tilde{C}_{x_j}, j \in \mathcal{J}} \quad R_\Sigma(\boldsymbol{\alpha}) \tag{14}$$

$$\text{s.t.} \quad \alpha_q R_\Sigma(\boldsymbol{\alpha}) \leq L_q, \quad \forall q \in \mathcal{J} \cup \{4\}, \tag{14a}$$

$$0 \leq C_{x_j} \leq P_j, \quad \forall j \in \mathcal{J}, \tag{14b}$$

$$|\tilde{C}_{x_j}|^2 \leq C_{x_j}^2, \quad \forall j \in \mathcal{J}, \tag{14c}$$

where, $\mathcal{J} = \{1, 2, 3\}$ is the set of all transmitters and $\alpha_4 = \alpha_1 + \alpha_2$ is defined in order to fit (12) into (13). The variables L_q, $\forall q$ are the functions of $C_{x_j}, \tilde{C}_{x_j}, j \in \mathcal{J}$ and are defined in (9)-(12). Transmission power is constrained by (14b), and the constraint (14c) ensures that the augmented covariance matrix is positive semidefinite [12].

Assuming that the optimal value of (14) is $R_\Sigma^*(\boldsymbol{\alpha})$ for a given $\boldsymbol{\alpha}$, the corresponding Pareto-optimal rate tuple is $\boldsymbol{\alpha} R_\Sigma^*$, which is the intersection of the rate region Pareto boundary with the ray in the direction of $\boldsymbol{\alpha}$.

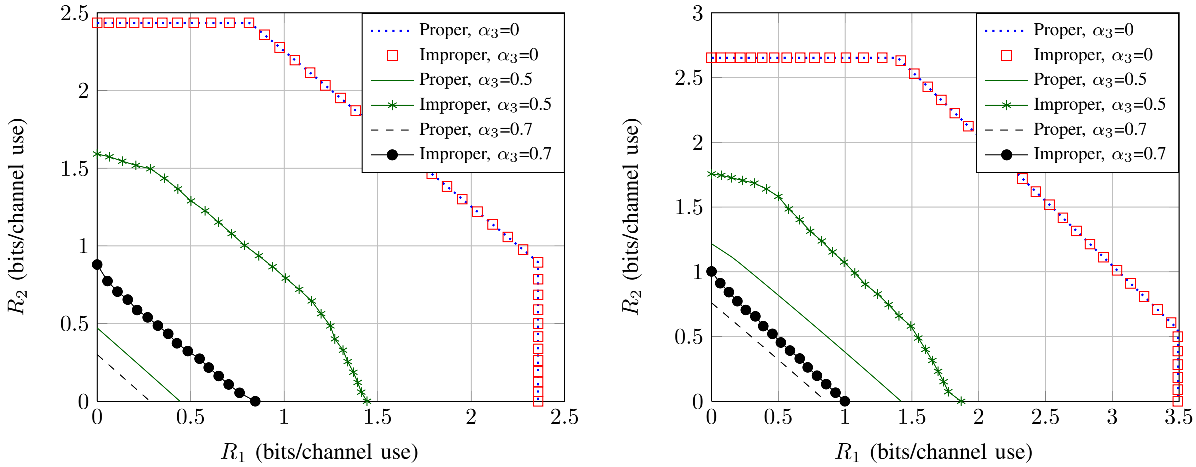
Merging the constraints (14a) into the objective function, problem (14) can be expressed as a maximization problem with a weighted Chebyshev objective function [22],

$$\max_{C_{x_j}, \tilde{C}_{x_j}, j \in \mathcal{J}} \ \min_{q \in \mathcal{J}} \frac{L_q}{\alpha_q} \tag{15}$$

$$\text{s.t.} \quad 0 \leq C_{x_j} \leq P_j, \quad \forall j \in \mathcal{J}, \tag{15a}$$

$$|\tilde{C}_{x_j}|^2 \leq C_{x_j}^2, \quad \forall j \in \mathcal{J}. \tag{15b}$$

Problem (15) is non-convex. This can be seen by replacing L_q, $\forall q$ with the expressions in (9)-(12). To obtain a reliable sub-optimal solution of this problem, it can be alternatively written as a SDP with rank constraints by means of some



(a) Channel realization H_1 (strong interference).

(b) Channel realization H_2 (weak interference).

Fig. 3: Comparison of improper and proper signaling at unit maximum transmit power and unit noise variance. The achievable rate region is depicted for the channel realizations H_1 and H_2 corresponding to strong and weak interference, respectively. α_3 is the target rate ratio of the P2P user.

vector definitions. The rank constraints are then relaxed to obtain a relaxed problem (SDR). The solution of the relaxed problem is then projected into the feasible set of the original problem. Details of this procedure are given in the arxiv version due to the page limit [23].

The resulting achievable rate region enjoys the benefits of improper signaling, in the form of an enlarged region in comparison with proper signaling. A numerical comparison is given in Section IV.

IV. Numerical Results

In this section, we examine the performance of the joint optimization procedure utilized for optimizing the variance and the pseudo-variance of the complex improper Gaussian signals. We consider two channel realizations in this section defined as (cf. (1) and (2))

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \end{bmatrix}.$$

Those channels are given by

$$\mathbf{H}_1 = \begin{bmatrix} 2.03e^{-i0.68} & 2.1e^{i2.64} & 3.2e^{i1.48} \\ 4.7e^{i1.97} & 4.5e^{-i0.66} & 2.85e^{i2.41} \end{bmatrix},$$
$$\mathbf{H}_2 = \begin{bmatrix} 3.2e^{-i0.72} & 2.3e^{i2.52} & 1.9e^{i1.35} \\ 2.8e^{i1.68} & 2.5e^{-i0.76} & 3.4e^{i2.23} \end{bmatrix}.$$

Note that $\mathbf{H}_1$ corresponds to a channel with strong interference, while $\mathbf{H}_2$ is a channel realization with weak interference.

We start by comparing the achievable rate regions using improper signaling in comparison to proper signaling. Fig. 3 compares those rate regions for the two given channel realizations. According to Fig. 3, in case of silent P2P communication ($\alpha_3 = 0$ in (13)), improper signaling does not enlarge the achievable rate region in comparison to proper signaling. This is due to the fact that proper signaling is optimal in the MAC, which coincides with the PIMAC with $\alpha_3 = 0$. For active P2P communication, improper signaling outperforms proper signaling from the rate region perspective. As can be seen from Fig. 3 with the corresponding channel realizations, stronger interference leads to higher gains by improper signaling. Here, we observes that, the advantage of improper Gaussian signaling gets more tangible by increasing the interference power. Intuitively, at high interference regime, interference alignment on single real dimension becomes feasible by improper Gaussian signaling. Hence, by sacrificing a real dimension for interference, single real streaming achieves an enlarged rate region. However, at low interference regime, unequal power allocation between real and imaginary components outperforms equal power allocation. This is due to the desired signal and interference shares at each real dimensions of the received complex signal. According to Fig. 4 reserving 50% of the network sum rate to the P2P communication, improper signaling improves the sum rate of the MAC users at least three times more than proper signaling considering strong interference channel realization.

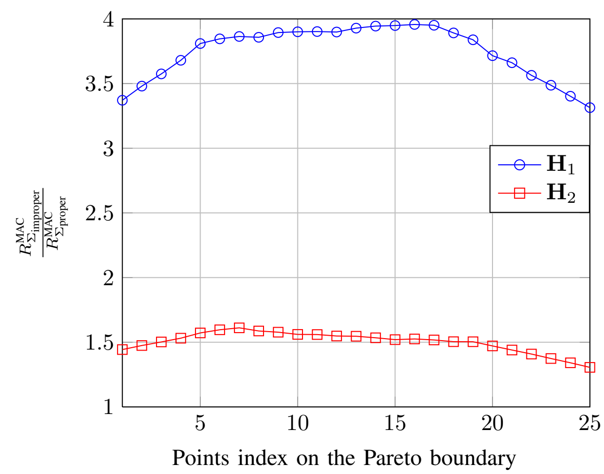


Fig. 4: Improvement in the sum rate of the MAC users by improper Gaussian signaling for $\alpha_3 = 0.5$.

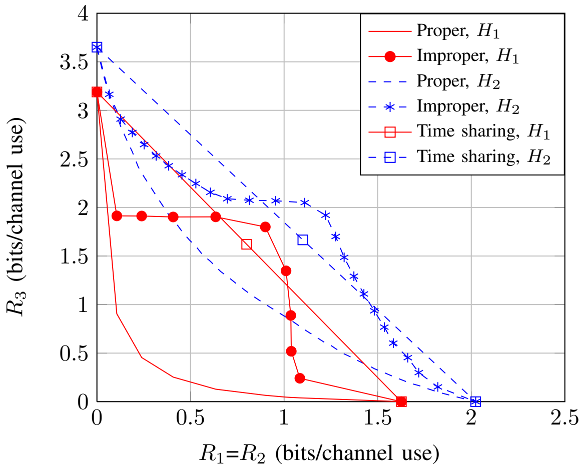


Fig. 5: Improvement of the achievable rate of the P2P user by improper signaling. Maximum transmit signal power and additive noise variance are set to unity.

Fig. 5 reflects the gain in R_3 achieved by improper signaling compared to proper signaling for equal transmission rate allocation for the users in the MAC. In this scenario, the P2P users can be viewed as an underlay cognitive radio which is activated if the demands of the primary system (MAC) is satisfied. According to this figure, reserving 50% of the overall sum rate to the MAC users, by improper Gaussian signaling the secondary users (P2P) can achieve higher rates compared to the case with proper Gaussian signaling. Furthermore, by proper Gaussian signaling, activating either MAC or P2P communication (time-sharing) achieves higher sum rate than the case when all users are active. However, switching the transmission to improper Gaussian, activating all users performs better for the given channel realizations. This can be seen from Fig. 5 where some rate tuples of improper Gaussian signaling are above the ones with time sharing strategy. This is an important observation towards capturing both fairness among users and sum rate optimality by utilizing improper Gaussian signaling.

V. Conclusion

We investigated the achievable rate region of the MAC in the presence of interference from a point-to-point (P2P) communication system sharing the same resources, using general (improper) Gaussian signaling. This P2P system might be an underlay cognitive radio, for instance. The achievable rate region is maximized with respect to the variance and pseudo-variance of the transmit signal, while treating interference as noise at the receivers. The benefit of using improper signaling is reflected by the fact that a non-zero pseudo-variance achieves a larger rate region in the MAC for a given rate of the P2P communication, and vice versa. Thanks to the extra degrees-of-freedom provided by non-zero pseudo-variance, which allows interference alignment over single real dimension at high interference regime. Moreover, we observed the benefits of improper Gaussian signaling in serving all users while still achieving sum-rate optimal points on the Pareto boundary.

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