Coordination gains in the cellular uplink with noisy interference

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Abstract—This paper studies the performance gains of coordination in an elemental cellular network consisting of a multiple-access channel (MAC) and a point-to-point channel (P2P) interfering with each other. It is assumed that this network, denoted as PIMAC, is operating in the noisy (very-weak) interference regime. Three schemes, denoted as naive-TIN, TDMA-TIN, and IA-TIN, each with different coordination requirements among the transmitters (and receivers), are compared with each other in terms of achievable sum rates. It is shown that, although the PIMAC is in the noisy interference regime, allowing more coordination between the users might increase the performance, depending on the channel parameters. Consequently, this proves the sub-optimality of TDMA-TIN and naive-TIN in those regimes, which is in contrast to existing results for $K$-user interference and X channels. The analytical finding are verified by numerical evaluations.

I. INTRODUCTION, NOTATION, AND SYSTEM MODEL

Communication in interference limited networks often requires some coordination between different nodes. This coordination can enhance the performance of a transmission scheme, at the expense of increasing its complexity and power consumption. However, communicating nodes can have some limitations in practice, such as limited computational capabilities or limited power. This calls upon simple schemes which are not demanding in terms of such resources.

Treating interference as noise (TIN) [1] is among the simplest schemes that can be used in interference networks. In its simplest form, TIN can be implemented as follows: transmitters send at their maximum allowed power all the time, and receivers ignore interference by treating it as extra noise. Note that this does not require any coordination between the transmitters, and does not increase the decoding complexity beyond the complexity of the interference-free decoders at the receivers. While this scheme might sound very naive at first, its optimality has been shown in several cases, such as the interference channel (IC) with noisy interference [2]–[5] and the X-channel with noisy interference [6].

The advantage of this naive-TIN scheme is that no coordination is required between cells in a cellular system. Coordination is needed when TIN is combined with power control as in [7], with time-division multiple-access (TDMA) as in [8] or with interference alignment (IA) as in [9], [10]. An interesting question is whether this additional coordination in cellular networks lead to improvements compared to naive-TIN even if the cellular interference networks exhibits noisy interference.

We address this question by studying a network consisting on a multiple-access channel (MAC) sharing the same medium with a point-to-point (P2P) channel. The resulting channel, denoted PIMAC (Fig. 1), has been studied earlier in [11]–[13]. In this paper, we focus on the following schemes:

- **Naive-TIN**: Transmitters send at full power and receivers treat interference as noise,
- **TDMA-TIN**: Tx1 and Tx3 (Rx1 as intended receiver) use the channel alternatively (TDMA with a time sharing parameter $\tau \in [0,1]$). Tx2 (Rx2 as intended receiver) sends continuously with full power, and the receivers treat interference as noise, and
- **IA-TIN**: Tx1 and Tx3 send private signals and alignment signals (interference alignment (IA) at Rx2), and Tx2 sends only private signals, and where the private signals are treated as noise at both receivers and the aligned interference is decoded at Rx2.

Note that these schemes are sorted in increasing order of complexity. These latter two schemes can be seen as improvements on naive-TIN which require more coordination and processing capabilities. We compare the performance of these schemes, and show that there exist regimes with very-weak interference where IA-TIN outperforms TDMA-TIN (which in turn outperforms naive-TIN as shown in [8]). This proves the sub-optimality of TDMA-TIN in these regimes.

A. Notation

We denote the Gaussian capacity function by $C(x) = \frac{1}{2} \log (1 + x)$ and use $C^+(x) = \max\{0, C(x)\}$. Scalars are denoted by normal fonts, and vectors by bold-face font.

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B. System Model

We consider a PIMAC as shown in Fig. 1, where Tx1 and Tx3 want to communicate with Rx1, and Tx2 wants to communicate with Rx2. Thus, the PIMAC consists of an underlying 2-user IC (consisting of Tx1, Tx2, Rx1, and Rx2) in addition to one extra transmitter (Tx3). It is known from [4] that TIN is optimal for the IC with noisy interference. To examine TIN in the PIMAC, we focus on the very-weak interference regime, i.e., where the interference is negligible compared to the desired signal.

Constants $d$ and $i$. The noise at Rx$_1$ is treated as noise. The constraint $P$ is symmetric as shown in Fig. 1.

The received signals of the two receivers at time index $t \in \{1, \ldots, n\}$ can be written as:

$$y_1[t] = h_d x_1[t] + h_c x_2[t] + h_\beta x_3[t] + z_1[t], \quad (1)$$
$$y_2[t] = h_c x_1[t] + h_d x_2[t] + h_\alpha x_3[t] + z_2[t], \quad (2)$$

where $x_i[t] \in \mathbb{R}, i \in \{1, 2, 3\}$ represents the transmit signal of Tx$i$ which has power $P_i$, satisfying the power constraint $P_i \leq P$, and $z_{j}[t], j \in \{1, 2\}$ is the Gaussian noise at Rx$j$, which has unit variance and is independently and identically distributed (i.i.d.) over time. The constants $h_d, h_c, h_\beta,$ and $h_\alpha$ represent the real (static) channel coefficients.

The sum-rate is defined as $R_\Sigma = R_1 + R_2 + R_3$, where $R_i$ represents the communication rate for Tx$i$. In this work, we focus on the very-weak interference regime, which we define as

$$\max\{Ph_\alpha^2, Ph_\beta^2\} < \sqrt{Ph_d^2} = \sqrt{\text{SNR}}. \quad (3)$$

In this regime, the power of interference is smaller than half the power of the desired signal on a logarithmic scale.

In the following sections, we present the different variants of TIN scheme.

II. NAIVE-TIN

Naive-TIN is the simplest variant of TIN in which the transmitters send their signals with the full power without using any coordination at all. At the receiver side, the users decode their desired signal as in the interference-free channel, i.e. by treating interference as noise. Rx1 decodes $x_1$ and $x_3$ as in the MAC while $x_2$ is treated as noise. Rx2 decodes its desired signal $x_2$ while the undesired signals $x_1$ and $x_3$ are treated as noise. The sum-rate achieved by naive-TIN is as follows.

$$R_{\Sigma, NT} = C \left( \frac{Ph_d^2}{Ph_\alpha^2 + 1} \right) + C \left( \frac{Ph_\beta^2}{Ph_\alpha^2 + 1} \right)$$

The first term in $R_{\Sigma, NT}$ is the achievable sum-rate for the MAC with noise variance $Ph_\alpha^2 + 1$ (treating $x_2$ as noise), and the second term represents the achievable rate for the P2P channel with the noise variance $P(h_\beta^2 + h_\alpha^2) + 1$ (treating $x_1$ and $x_3$ as noise).

III. TDMA-TIN

Consider the case when $h_\beta^2 < h_\alpha^2$ and $h_\alpha^2 < h_\beta^2$. In this case, Tx3 can send more information to Rx1 than Tx1 while causing less interference at Rx2. Thus, it would be better to switch Tx1 off and operate the PIMAC as an IC with Tx2 and Tx3 active. The intuition from this example leads us to a smarter variant of TIN, namely TDMA-TIN. This scheme is a combination of TDMA and TIN in which Tx1 transmits with the power $P/\tau$ in a fraction $\tau \in [0, 1]$ of time, and Tx3 transmits in the remaining fraction $1 - \tau$ of the time with the power $P/(1 - \tau)$. Using this scheme, we have two 2-user IC’s operating over orthogonal time slots. The complexity of this scheme is slightly higher than that of naive-TIN. Namely, in order to establish the coordination between Tx1 and Tx3, Rx1 has to provide the Tx’s with the time schedule. No additional complexity is required since Tx2 sends as in the interference-free P2P channel and both Rx’s treat interference as noise. By using this scheme, the sum-rate given in (4) at the top of the next page can be achieved.

IV. IA-TIN

This scheme is a combination of interference alignment and TIN, which might be more capable than naive-TIN and TDMA-TIN. In this scheme, Tx1 and Tx3 do not only send private signals which are treated as noise at Rx2, but they also send alignment signals which are aligned at Rx2 while received separately at Rx1. Therefore, Rx1 is able to decode both signals individually while Rx2 decodes their linear combination. This is similar to common signal decoding in the Han-Kobayashi scheme [14] with the main difference that the two interferers are treated by Rx2 as one interferer (due to alignment). Since Rx2 must be able to decode a linear combination of the aligned signals, we require a code with a linear structure such as nested-lattice code. Note that lattice codes for interference alignment have been introduced in [15] for the many-to-one interference.
channel (M2O-IC). However, in contrast to the M2O-IC, in the PIMAC, we have to ensure that the signals are aligned at the undesired receiver, while separable at the desired receiver. In what follows, we describe the scheme in more detail.

A. Encoding:
In this scheme, Tx\(i\), \(i \in \{1, 3\}\) sends the following signal
\[
x_i = x_{i,p} + x_{i,a}, \quad i \in \{1, 3\},
\]  
(5)
where \(x_{i,p}\) and \(x_{i,a}\) denote the private and the alignment signals with powers \(P_{i,p}\) and \(P_{i,a}\), respectively, satisfying \(P_{i,a} + P_{i,p} = P_i \leq P\). The private signal \(x_{i,p}\) is Gaussian-distributed. To generate the alignment signals, both transmitters (Tx1 and Tx3) use the same \(n\)-dimensional nested-lattice codebook [16], [17] with fine lattice \(\Lambda_f\), coarse lattice \(\Lambda_c\), rate \(R_a\), and power 1. The alignment signal \(x_{i,a}\) is a symbol or element of \(x_{i,a}\) (of length \(n\)) which is given by
\[
x_{i,a} = \sqrt{P_{i,a}}[(a_i - d_i) \mod \Lambda_c],
\]  
(6)
where \(a \) is a point in the nested-lattice codebook and \(d_i\) is a random dither [16] which is known at both receivers.

Since the alignment signals from Tx1 and Tx3 have to be aligned at Rx2, their powers have to fulfill the following condition
\[
P_{1,a}h_c^2 = P_{3,a}h_c^2.
\]  
(7)
Moreover, since the private signals \(x_{1,p}\) and \(x_{3,p}\) need to be decoded only at the desired receiver, i.e., Rx1, we assign their power such that they are received under the noise level at Rx2. Therefore, we obtain
\[
P_{1,p} = \frac{1}{h_c^2}, \quad P_{3,p} = \frac{1}{h_a^2}.
\]  
(8)
Now, consider Tx2. This transmitter sends
\[
x_2 = x_{2,p1} + x_{2,p2},
\]  
(9)
where \(x_{2,p1}\) and \(x_{2,p2}\) are Gaussian distributed private signals with the powers \(P_{2,p1}\) and \(P_{2,p2}\), respectively, satisfying \(P_{2,p1} + P_{2,p2} = P_2 \leq P\). Similar to the private signals of Tx1 and Tx3, the private signals from Tx2 have to be received under the noise level at Rx1. Therefore, we can write
\[
P_{2,p1} + P_{2,p2} \leq \frac{1}{h_c^2}.
\]  
(10)
Note that compared to the previous schemes, IA-TIN needs more coordination since perfect CSI is needed at the Tx’s for power allocation. This increases the complexity of the scheme.

B. Decoding
Since the PIMAC is not symmetric, the decoding process is not the same for both receivers. Therefore, we discuss the decoding at the Rx’s separately.

1) Decoding at Rx1: Rx1 starts with decoding the alignment signals while treating the private signals as noise. The decoding order of the alignment signals depends on the channel strength. If Rx1 decodes \(x_{3,a}\) first and then \(x_{1,a}\), then the rate \(R_a\) has to satisfy
\[
R_a \leq \min\{R_{a,1}^{[1]}, R_{a,3}^{[1]}\},
\]
\(\text{where}\)
\[
R_{a,1}^{[1]} = C \left(\frac{h_c^2 P_{1,a}}{1 + h_a^2 P_{3,a} + h_c^2 P_{3,p} + h_a^2 P_2}\right),
\]
\[
R_{a,3}^{[1]} = C \left(\frac{h_a^2 P_{3,a}}{1 + h_a^2 P_{3,1} + h_a^2 P_{3,p} + h_a^2 P_2}\right).
\]
In the other decoding order, the rate \(R_a\) has to satisfy
\[
R_a \leq \min\{R_{a,1}^{[2]}, R_{a,3}^{[2]}\},
\]
\(\text{where}\)
\[
R_{a,1}^{[2]} = C \left(\frac{h_a^2 P_{1,a}}{1 + h_a^2 P_{3,p} + h_a^2 P_3 + h_a^2 P_2}\right),
\]
\[
R_{a,3}^{[2]} = C \left(\frac{h_a^2 P_{3,a}}{1 + h_a^2 P_{3,1} + h_a^2 P_{3,p} + h_a^2 P_2}\right).
\]
Rx1 chooses the best decoding order, leading to a rate constraint
\[
R_a \leq \max\{\min\{R_{a,1}^{[1]}, R_{a,3}^{[1]}\}, \min\{R_{a,1}^{[2]}, R_{a,3}^{[2]}\}\}
\]  
(11)
The remaining desired signals \(x_{1,p}\) and \(x_{3,p}\) are treated the same for both cases. To this end, Rx1 decodes \(x_{1,p}\) and \(x_{3,p}\) as in a MAC while treating \(x_{2,p1}\) and \(x_{2,p2}\) as noise. Therefore, the following sum of \(R_{1,p}\) and \(R_{3,p}\) can be achieved
\[
R_{3,p} + R_{1,p} = C \left(\frac{h_c^2 P_{3,p} + h_a^2 P_{1,p}}{1 + h_a^2 P_2}\right).
\]  
(12)\(^2\)
\(^2\)Nested-lattice codes achieve the capacity of the point-to-point AWGN channel [17].
2) Decoding at Rx2: The decoding order at Rx2 is \( x_{2,p1} \to (a_1 + a_3) \mod \Lambda_c \to x_{2,p2} \). First, the receiver decodes \( x_{2,p1} \) while treating the other signals as noise. Thus, the following rate is achievable

\[
R_{2,p1} = C \left( \frac{h^2_{a} P_{2,p1}}{1 + h^2_{a} P_{1,p} + h^2_{a} P_{3,p} + h^2_{a} P_{2,p2}} \right). \tag{13}
\]

After removing the contribution of \( x_{2,p1} \) from \( y_2 \), Rx2 decodes the sum \( (a_1 + a_3) \mod \Lambda_c \). Decoding this sum is possible as long as \[16\]

\[
R_a \leq C^+ \left( \frac{h^2_{a} P_{1,a}}{1 + h^2_{a} P_{1,p} + h^2_{a} P_{3,p} + h^2_{a} P_{2,p2}} - \frac{1}{2} \right). \tag{14}
\]

The receiver can then construct the received sum of alignment signals \( h_c x_{1,a} + h_{\alpha} x_{3,a} \) from the decoded sums of codewords \( (a_1 + a_3) \mod \Lambda_c \) (cf. [18]). After reconstructing the sum of alignment signals, it is subtracted from the received signal (in addition to the contribution of \( x_{2,p1} \)) and then \( x_{2,p2} \) is decoded. Decoding \( x_{2,p2} \) is possible reliably with the following rate

\[
R_{2,p2} = C \left( \frac{h^2_{a} P_{2,p2}}{1 + h^2_{a} P_{1,p} + h^2_{a} P_{3,p}} \right). \tag{15}
\]

Therefore, the sum-rate given by

\[
R_{\Sigma,TT} = 2R_a + R_{1,p} + R_{2,p1} + R_{2,p2} + R_{3,p}, \tag{16}
\]

where the summands above satisfy (11)-(15). In the next section, we compare the performance of different types of TIN. Note that in [8], it is shown that the naive-TIN is outperformed by TDMA-TIN\(^3\).

V. PERFORMANCE COMPARISON

In this section, we want to compare the performance of IA-TIN with TDMA-TIN and naive-TIN analytically.

Now, let us compare the performance of IA-TIN and TDMA-TIN. For convenience, we define the following quantities

\[
m_c = \frac{1}{2} \log_2(P h^2_{a}) \quad \kappa \in \{d,c,\beta,\alpha\}. \tag{17}
\]

Let \( R_{IA} \) denote a regime in which the channels satisfy (3) in addition to \( m_\beta \in \left[ \frac{m_c}{\alpha}, \frac{m_c}{\beta} \right] \), where

\[
m_\beta = \max \left\{ m_d - m_c, m_d + m_\alpha - 2m_c \right\}, \tag{18}
\]

\[
m_\alpha = \min \left\{ m_\alpha, m_d, m_d - m_c + 2m_\alpha \right\}. \tag{19}
\]

The goal is to show that in regime \( R_{IA} \) and for sufficiently high SNR (SNR >> 1), there exists power allocation for IA-TIN such that TDMA-TIN is outperformed. To show this, we start by identifying the achievable sum-rate of TDMA-TIN at high SNR. Applying \( m_c < m_d - m_c < m_\beta \) and \( m_\alpha < m_d \) (conditions of regime \( R_{IA} \)), to (4) in high SNR regime, the achievable sum-rate of the TDMA-TIN can be approximated as

\[
R_{\Sigma,TT} = \max_{\tau \in [0,1]} \left\{ \left(1 - \tau\right)\left[m_\beta - m_c + m_d - m_\alpha\right] \right. \\
+ \left. \tau\left[2(m_d - m_c)\right] \right\}. \tag{20}
\]

Since this maximization is linear in \( \tau \), the optimal value of \( \tau \) must be in \( \{0,1\} \). Therefore, the achievable sum-rate of TDMA-TIN at high SNR regimes under the conditions given in (18) and (19) is given by

\[
R_{\Sigma,TT} = m_d - m_c + \max\{m_d - m_c, m_\beta - m_\alpha\}. \tag{21}
\]

Next, we give the power allocation of IA-TIN such that IA-TIN outperforms TDMA-TIN at high SNR. The power allocation for the private signals of Tx1 and Tx3 are given in (8). To allocate the remaining powers, we need to split the whole regime \( R_{IA} \) into four sub-regimes. Here, we consider the power allocation for the sub-regime, which is identified as follows

\[
m_\alpha < m_c, \quad m_d - m_c < m_\beta - m_\alpha. \tag{21}
\]

In this case, the allocated powers are given by

\[
P_{2,p2} = \max \left\{ P h^4_{a}, h^2_{c} h^2_{\alpha}/h^2_{\beta} \right\}, \tag{22}
\]

\[
P_{2,p1} = \frac{1}{h^2_{c}} - P_{2,p2} \tag{23}
\]

\[
P_{3,a} = P - h^2_{P_{2,p2}}. \tag{24}
\]

\( P_{1,a} \) is obtained by substituting (24) into (7). It can be verified that the assigned powers satisfy the power constraint. By substituting these power allocation parameters into the rate constraints given in (11)-(15), and letting SNR \( \rightarrow \infty \), we obtain the achievable rates

\[
R_{3,p} + R_{1,p} \leq m_\beta - m_\alpha, \tag{25}
\]

\[
R_{2,p1} \leq m_d - m_c - m_\alpha, \tag{26}
\]

\[
R_{2,p2} \leq \max\{m_\beta + m_c - m_d - m_\alpha, 2m_\alpha - m_\beta - m_c + m_d\}, \tag{27}
\]

\[
R_a \leq \min\{2m_\alpha + m_\beta - m_c - m_\alpha, m_\beta + m_c - m_d - m_\alpha\}. \tag{28}
\]

By summing up these achievable rates we get

\[
R_{\Sigma,IT} = 2R_a + R_{1,p} + R_{3,p} + R_{2,p1} + R_{2,p2} \tag{29}
\]

which is larger than \( R_{\Sigma,TT} \) since \( R_a \) is strictly positive (19), (21) (except if \( m_\beta - m_\alpha = m_d - m_c \)). As a conclusion, in regime \( R_{IA} \), the IA-TIN scheme with the given power allocations strictly outperforms TDMA-TIN at high SNR if (21) is satisfied. The same behavior can be shown similarly for the remaining cases of regime \( R_{IA} \), with details given in [19].
Fig. 2: Normalized achievable sum-rate as a function of \( m_\beta \) for a PIMAC with \( h_d = 1, P = 40\, \text{dB}, m_\alpha = 0.45m_d \) and \( m_c = 0.5m_d \).

Fig. 3: Achievable sum-rate versus SNR for a PIMAC with \( h_d = 1, m_\alpha = 0.45m_d, m_c = 0.5m_d, \) and \( m_\beta = 1.2m_d \).

VI. DISCUSSION

The following figures show numerical comparison of the achievable rates of the TIN schemes with a sum-rate upper bound obtained from [8]. Fig. 2 shows the normalized achievable sum-rate \( R_S/m_d \) as a function of \( m_\beta \) for a PIMAC with \( h_d = 1, P = 40\, \text{dB}, m_\alpha = 0.45m_d \) and \( m_c = 0.5m_d \). Such a PIMAC has very-weak interference (it satisfies 3). For this PIMAC, regime \( R_{IA} \) corresponds to \( m_\beta/m_d \in [0.5, 1.4] \). It can be seen from the figure that IA-TIN outperforms TDMA-TIN in some subsets of this interval. While it is shown in Section V that IA-TIN outperforms TDMA-TIN for the whole \( R_{IA} \) regime at asymptotically high SNR, this figure shows that this effect starts to take place at lower SNR values. In this example, although the PIMAC has very-weak interference, neither naive-TIN nor TDMA-TIN can be optimal.

Figure 3 shows the achievable sum-rate versus SNR for a PIMAC with \( h_d = 1, m_\alpha = 0.45m_d, m_c = 0.5m_d, \) and \( m_\beta = 1.2m_d \). From this figure, it can be seen that IA-TIN starts to outperform TDMA-TIN at an SNR of 30 dB and has the same slope as the upper bound at high SNR, which indicates that it is asymptotically optimal (within a constant gap).

REFERENCES