# Energetically Consistent Modeling of Passive Memelements

Karlheinz Ochs, Enver Solan*

*Chair of Digital Communication Systems,*
*Electrical Engineering and Information Technology,*
*Ruhr-Universitaet Bochum, 44780 Bochum, Germany*

---

**Abstract**

Memory elements are suited for building self-organizing circuits. In contrast to memristive devices (nonlinear resistors with memory), memreactive devices (nonlinear capacitors or inductors with memory) are lossless and can be utilized in order to achieve more degrees of freedom in electrical circuits with respect to adaptively adjustable parameters.

Fabrication of pure memreactive elements is hard yet and hence mathematical models are needed to make pioneering pre-investigations. Modeling of memreactive elements is closely related to the modeling of nonlinear reactive elements. Common memreactive models are based on a static definition. Thus, they are not passive in general. But losslessness of such devices is of great importance from a circuit theoretic point of view. We propose a modeling approach for lossless memreactive elements based on an energetic definition. In this context, a novel equivalent circuit of memory devices including memtransformer or memgyrator is introduced. A known meminductive model is utilized in simulations and comparisons between different definitions are shown in order to underly the necessity of an energetically consistent model.

*Keywords:* memristive devices, memgyrator, memtransformer, memcapacitor, meminductor

*2010 MSC:* 00-01, 99-00

*Corresponding author
*Email address:* Enver.Solan@rub.de (Enver Solan)

## 1. Introduction

Electrical circuits with adaptively adjustable parameters are used in order to preserve a desired functionality under variable environmental conditions. Adaptive filters, as general signal processing blocks, are popular examples of such systems. Whenever a filter's parameters must track time-varying conditions, adaptive filters are attractive options. In particular, in high-speed applications adaptive analog filters are more important than their digital equivalents. These filters mostly are realized by exploiting an error minimizing feedback-loop [1]. The adaption in such systems appears in an instantaneous regulation of the error, where the desired behavior should be known a priori. We intend to use adjustable elementary circuit elements with memory in order to achieve a similar functionality in a more efficient manner. An innovative example of such an element is the memristor, also referred to as memristive device or system, which is essentially a nonlinear resistor with memory [2]. The memory effect of these devices makes them suitable for an adaption of circuit parameters but in a self-organizing manner. This means, that the parameter adjustment is associated to a kind of a learning behavior depending on environmental experiences done in the past. With this, the adaption is more efficient and is not only related to an instantaneous reaction to environmental changes. Popular examples, where these elements have been used are neuromorphic circuits [3, 4]. There, the memristive devices replace synapses in order to interconnect neurons. A more general approach is the interconnection of oscillators through memristors [5, 6]. Moreover, an adaptively adjustable oscillator circuit including memristors and capable of anticipation of general information has been presented in [7, 8].

Memristors are elements with losses and thus they belong to the class of dissipative systems. Considering electrical filters or oscillators, memristors can be utilized to adapt resistive parameters. Hence, the quality factor of an oscillator, for example, can be adjusted with such elements, whereas the resonance frequency is only marginally influenced. In order to achieve more degrees of freedom, lossless reactive elements with memory - memcapacitances or meminductances - are preferable. Some applications of memreactive elements can already be found in the literature, e.g. in the context of memcomputing or in neural networks [9].

Unfortunately, the fabrication of pure memreactive elements is hard yet. Memreactive behavior has been observed mostly as parasitic effects in memristive devices [10, 11, 12]. Therefore, mathematical models of memreactive elements are even more important in order to make pre-investigations of circuits including them before fabrication. A lot of mathematical models of memreactive elements, namely memcapacitances as well as meminductancens, are available [13, 14, 15, 16]. Another approach regarding memreactive models is about mixed signal circuits consisting of microcontroller and operational amplifier imitating memreactive behavior [17].

Common existing mathematical models are based on a static definition of nonlinear reactive one-ports and hence they are in general not passive. Depending on incorporated parameters and the utilized excitation, an active behavior of such models can be observed [13], which is unfortunate from a circuit theoretic point of view. By definition, these elements should be even lossless. In particular, in the context of memristive or memreactive emulators an energetically consistent model is unavoidable [18, 19, 20, 21]. We are intended to find an energetically consistent modeling approach for passive memristive and lossless memreactive elements independently of concrete model parameters and excitations. Although evaluating the passivity of memristive systems is a trivial task by inspecting the memristance, starting our proposed approach with these elements facilitates the accessibility of reactive devices with memory.

Modeling of memreactive elements is closely related to the modeling of nonlinear reactive elements. It is known that a static definition of nonlinear reactive elements leads to non-passive models, whereas an energetic definition yields energetically consistent models with respect to positive device values [22]. Those definitions depend on a suitable choice of complimentary constitutive variables. In accordance with the definition of general memristors, we define an $n$th-order

$x$-controlled memory device by

$$y(t) = h\left(\boldsymbol{z}, x, t\right) x(t)\,, \quad \dot{\boldsymbol{z}} = \boldsymbol{f}\left(\boldsymbol{z}, x, t\right)\,, \tag{1}$$

where the generalized response $h$ relates the input and output signal $x$ and $y$, respectively. The differential equation with the $n$-dimensional memory function $\boldsymbol{f}$ forms the memory of this device having an $n$-dimensional state vector $\boldsymbol{z}$. Depending on the device, $x$ and $y$ must be chosen appropriately such that they are complementary constitutive variables. This aspect will be consolidated in detail later on. We propose the adaption of the insights with respect to modeling nonlinear reactive elements to the modeling approach of memreactive elements. An energetic definition of memreactive elements will turn out in a parametric representation of the memory with energy-neutral two-ports, namely memtransformer and memgyrator [23, 24]. First investigations of energetically consistent memcapacitive models can be found in [25]. Since the correct evaluation of the hysteresis has to be reformulated considering the proposed novel approach, a paradigm change in modeling of memreactive elements has been introduced with this paper.

The paper is organized as follows: In the next Sec. 2, the mathematical model of a general memristor is recapitulated. In this context, energy-neutral two-ports like memtransformer and memgyrator are introduced in order to get an alternative electrical interpretation of the memory effect. Afterwards, in Sec. 3 different modeling approaches of memreactive elements including energetic, static, and differential definitions are presented. Simulation results show the differences between those definitions and the importance of an energetically consistent model. Main results of the work are summarized in the conclusion at the end of the manuscript.

## 2. Memristor

In general, memristors are nonlinear resistors with memory. A state-dependent resistance value of such elements leads to adjustable resistive parameters in electrical circuits, which are in particular beneficial in self-organizing systems [3, 7, 8]. The corresponding dual elements are memductors. In the next paragraph, the mathematical definition of those elements is recapitulated. In this context, a novel electrical representation of such devices based on transformers and gyrators with memory is introduced.

*2.1. Memristance*

The $n$th-order $i$-controlled memristive system [2]

$$u(t) = \hat{R}\left(\boldsymbol{z}, i, t\right) i(t) \,, \quad \dot{\boldsymbol{z}}(t) = \boldsymbol{f}\left(\boldsymbol{z}, i, t\right) i(t) \,, \tag{2}$$

with memristance $\hat{R}$ as the general response and memory function $\boldsymbol{f}$, is a generic description of a memristor. The current $i$ and voltage $u$ are defined as the input and output variables, respectively. The appropriate choice of these variables plays a key role in modeling reactive devices with memory, as shown later in the manuscript. Memristors are dissipative systems except for the special cases where the memristor degenerates to a short-circuit or to an open-circuit, and hence it is passive for $\hat{R} \geq 0$. We intend to find a parametric representation of passive memristive systems that is as simple as possible. To this end, the memory effect of these devices are incorporated into the transmission ratio of an ideal transformer

$$n = \hat{n}\left(\boldsymbol{z}, i, t\right) \,. \tag{3}$$

This leads to ideal transformers with memory - memtransformers. The memtransformer

$$u(t) = \hat{n}\left(\boldsymbol{z}, i, t\right) v(t) \,, \quad j(t) = \hat{n}\left(\boldsymbol{z}, i, t\right) i(t) \,, \tag{4}$$

where the voltage $u$ and current $i$ with respect to the primary side are interrelated to the voltage $v$ and current $j$ of the secondary side via the transformation ratio $\hat{n}$, is energy-neutral, independently of the transformation ratio, cf. Fig. 1 (middle). This can be verified from the instantaneous power transport of the two-port

$$p(t) = u(t)i(t) - v(t)j(t) = 0 \,. \tag{5}$$

Describing the memristance in terms of the state-dependent transmission ratio

$$\hat{R}(\boldsymbol{z}, i, t) = \hat{n}^2(\boldsymbol{z}, i, t)\, R \tag{6}$$

results in the equivalent circuit of Fig. 1 (middle). Here, the memory effect of

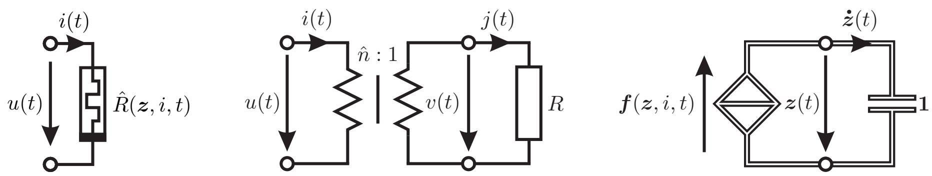


Figure 1: Electrical representation of an $n$th-order $i$-controlled memristive system. Memristive one-port (left), equivalent circuit with memtransformer (middle), and electrical interpretation of the memory (right).

the system has been considered by an integrator circuit consisting of an ideal current source interconnected to a capacitor, cf. Fig. 1 (right). In general, such devices could have more than one internal state. Therefore, the integrator circuit is denoted as a multi-port. Since the memtransformer is energy-neutral independently of the transmission ratio and it is port-wise interconnected with a constant resistor $R$, the whole system is passive if and only if $R \geq 0$. In case of $\hat{n} = 0$, the primary side of the memtransformer degenerates to a short-circuit, whereas the secondary side becomes to an open-circuit and hence passivity is still guaranteed. Consequently, regarding the alternative electrical representation, it suffices to examine the value of the constant resistor in order to evaluate if a memristive system is passive or not. The benefits of such an electrical representation become even more obvious in case of reactive elements with memory.

### *2.2. Memductance*

The dual element of a memristor is the memductor. Corresponding to a memristive system, an $n$th-order $u$-controlled memductive system

$$i(t) = \hat{G}(\boldsymbol{z}, u, t)\, u(t)\,, \quad \dot{\boldsymbol{z}}(t) = \boldsymbol{f}(\boldsymbol{z}, u, t)\, u(t) \tag{7}$$

is described by its memductance $\hat{G}$ instead of the memristance $\hat{R}$. Subject to memristive systems, passivity for memductive systems is ensured for $\hat{G} \geq 0$.

In case of memductive systems a parametric representation by a gyrator with memory - memgyrator - is intended. Therefore, a state-dependent gyration resistance is introduced

$$\hat{r} = \hat{r}\,(\boldsymbol{z}, u, t)\ . \tag{8}$$

Considering the instantaneous power transport of a memgyrator

$$\begin{aligned} &p(t) = u(t)i(t) - v(t)j(t) = 0\,, \quad \text{with} \\ &u(t) = \hat{r}\,(\boldsymbol{z}, u, t)\, j(t)\,, \quad v(t) = \hat{r}\,(\boldsymbol{z}, u, t)\, i(t)\,, \end{aligned} \tag{9}$$

with $u$, $i$, and $v$, $j$ as voltages and currents at the primary and secondary side of the memgyrator, respectively, it is clear, that the memgyrator is energy-neutral for arbitrary gyration resistances, cf. Fig. 2. Redefining the memductance in terms of the gyration resistance

$$\hat{G}\,(\boldsymbol{z}, u, t) = R/\hat{r}^2\,(\boldsymbol{z}, u, t) \tag{10}$$

leads to the electrical representation shown in Fig. 2 (middle) with a constant resistor $R$. Referring to the memtransformer, the memgyrator is energy-neutral

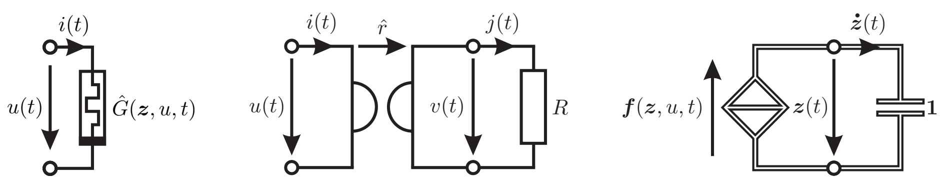


Figure 2: Electrical representation of an $n$th-order $u$-controlled memductive system. Memductive one-port (left), equivalent circuit with memgyrator (middle), and electrical interpretation of the memory (right).

independently of the gyration resistance $\hat{r}$ and it is interconnected to a constant resistor $R$. Therefore, the memductive system is passive if and only if $R \geq 0$. With this, passivity can be obviously evaluated by structural information of the equivalent circuit.

## 3. Reactive Elements with Memory

The modeling of reactive elements with memory is closely related to the modeling of nonlinear reactive elements. Because of the time dependencies of the parameters, there are several possibilities to extend the equation for constant device values, like energetic [22], static [13], and differential [26] reactive elements. Especially, the energy of these reactive elements with memory is inspected in order to derive conditions for passivity. In the sequel, a consistent modeling approach of lossless memreactive elements based on the utilization of memtransformers as well as memgyrators is introduced.

### *3.1. Meminductive One-Ports*

As a starting point, the modeling approach of a meminductive system is shown. The importance of a consistent model is emphasized by comparisons of simulation results based on an energetic, static and differential definition of meminductive systems with respect to different input signals. In order to get comparable results, a known meminductive model from the literature has been exploited in the simulations [15]. The mathematical model of the meminductive system

$$\hat{L}\left(q\left(t\right)\right) = L_0 + \frac{L_1 - L_0}{a e^{-4\gamma q(t)} + 1} \;, \quad a = \frac{L_1 - \hat{L}(0)}{\hat{L}(0) - L_0} \;, \quad q(t_0) = 0 \qquad (11)$$

includes several parameters like the lower/upper bound of the meminductance $L_0/L_1$, a parameter $a$ comprising the initial value $\hat{L}(0)$, as well as a material parameter $\gamma$. Utilized parameter values are $L_0 = 1$ mH, $L_1 = 10$ mH, $\gamma = 10$ 1/[mAs], and $L(0) = 2$ mH [15]. The starting time is denoted by $t_0 = 0$. Fig. 3 depicts the simulation scenario, where the meminductance is directly interconnected to a resistive voltage source. Two different excitation signals have been exploited in order to verify the differences between varying modeling approaches. Firstly, the meminductive one-port is excited by a sinusoidal input signal

$$e_0(t) = A_0 \sin\left(\Omega_0 t\right) \;, \quad \text{with} \quad \Omega_0 T_0 = 2\pi \;. \qquad (12)$$

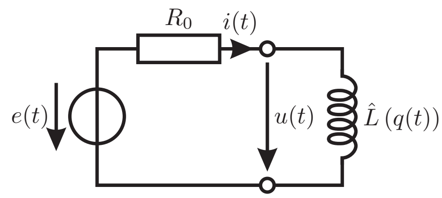


Figure 3: Electrical circuit representing the simulation scenario with internal source resistance $R_0 = 100\Omega$.

This excitation has been utilized for generating the hysteresis curves, which are distinctive features of devices with memory. Secondly, the meminductive system is excited by a sinusoidal input signal

$$e_1(t) = e_0(t) + \frac{A_1}{1 + e^{k[t-3T_0]}} + A_2 \tag{13}$$

with a negative offset $A_1 + A_2$ for three periods and with a positive offset $A_2$ for another three periods. Input signals with offsets have been exploited in order to inspect the stored energy within the device which is an indicator of passivity.

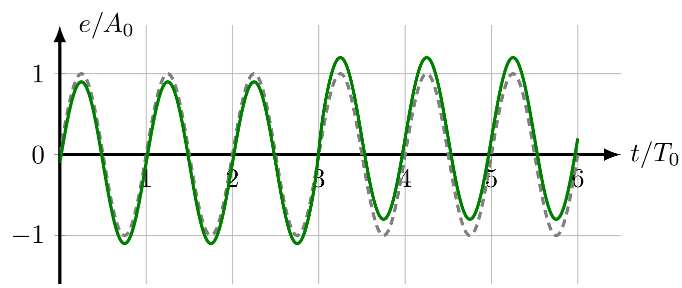


Figure 4: Two excitation signals used in the simulations. On the one hand the sinusoidal excitation without offset (gray-dashed) and on the other hand the sinusoidal signal with negative and positive offset (green). Inserted parameters values are $A_0 = 0,5$ V, $A_1 = -0,15$ V, $A_2 = 0,1$ V, $k = 100$ Hz, and $T_0 = 100$ ms.

*Energetic Meminductive System.* Common meminductive models existing in the literature are based on a static definition and hence they are not necessarily lossless, let alone passive [13]. This is related to an inappropriate assumption of complementary constitutive variables. We are intended to introduce a model, which preserves the losslessness of reactive devices with memory. Therefore, the passivity is defined in terms of the stored energy within the device

$$E(t) = \int_{t_0}^{t} u(\tau)i(\tau)\mathrm{d}\tau \geq 0 \,, \quad E(t_0) = 0 \,, \tag{14}$$

which has to be positive if the initially stored energy is zero. Starting from this definition, we use the energy-current relationship as a description of the meminductive system

$$E(t) = \frac{1}{2}\hat{L}\left(\boldsymbol{z}, i, t\right) i^2(t) \,. \tag{15}$$

With this, the current to the square $i^2$ can be identified as the input signal, whereas the stored energy is the output signal and $\hat{L}/2$ represents the general response of the dynamical system, cf. Equation 1. Since the current to the square is always positive $i^2 \geq 0$, the meminductive system is passive for $\hat{L} \geq 0$.

Similar to memristive systems, an equivalent circuit including a memtransformer or memgyrator representing the parametric effects is intended. In order to get such an equivalent circuit, the voltage-current relationship has to be figured out. Using the instantaneous power

$$p(t) = i(t)u_\mathrm{e}(t) = \frac{\mathrm{d}E(t)}{\mathrm{d}t} \,, \tag{16}$$

the voltage-current relationship can be described by

$$u_\mathrm{e}(t) = \sqrt{\hat{L}}\frac{\mathrm{d}\sqrt{\hat{L}}i(t)}{\mathrm{d}t} \,. \tag{17}$$

Here, $u_\mathrm{e}$ indicates the voltage at the energetic meminductive one-port. This relationship leads to the corresponding equivalent circuit in Fig. 5 (middle) with a memtransformer for $\hat{L} = \hat{n}^2 L$ and to the dual equivalent circuit in Fig. 5 (right) including a memgyrator for $\hat{L} = \hat{r}^2 C$, where $L$ and $C$ are constant inductances and capacitances, respectively. Since the energetic definition yields a port-wise interconnection of energy-neutral two-ports and a lossless one-port, the resulting meminductive one-port is lossless, if and only if, $L \geq 0$ and $C \geq 0$, respectively. Therefore, the losslessness can be evaluated by inspecting the sign of the constant inductance or capacitance.

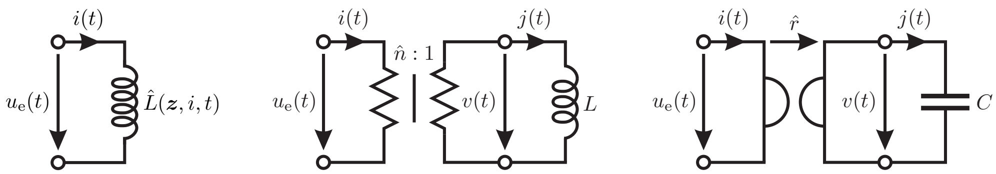


Figure 5: Meminductive one-port (left), corresponding electrical representation of a lossless meminductive system with a memtransformer (middle), and the dual circuit including a memgyrator (right)

*Simulation Results.* Simulation results of the energetic meminductance model are shown in Fig. 6. Considering the input-output relation of an energetic me-

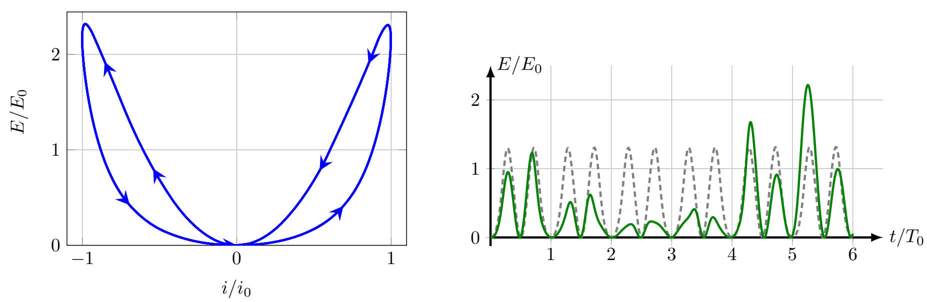


Figure 6: Hysteresis of the meminductive system based on an energetic definition (left) and the stored energy within the device (right) for an excitation without (gray-dashed) and with offset (green). Normalization values are $E_0 = 0,45$ nJ, $i_0 = 5$ mA, and $T_0 = 100$ ms.

minductive system, the hysteresis should be evaluated in the $E$-$i$-plane, which is shown on the left-hand side of Fig. 6, cf. Equations 1 and 15. The stored energy of the device is shown on the right-hand side of Fig. 6. It should be emphasized, that the stored energy within the device is always positive, independently of the excitation and model parameters. Therefore, passivity is ensured for an energetic definition of a meminductive system.

*Static Meminductive System.* The most meminductive models in the literature are based on a static definition [13, 14, 15], with the input-output relation

$$\varphi(t) = \hat{L}(\boldsymbol{z}, i, t)i(t)\,. \tag{18}$$

Here, the magnetic flux is the output variable, whereas the current describes the input variable. The meminductive model itself $\hat{L}$ stands for the generalized response of the system. Meminductive systems based on a static definition are not necessarily passive [13]. This fact becomes more clear when the voltage-current relationship is considered. It follows from Equation 18

$$\varphi(t) = \hat{L}i(t) \Leftrightarrow u_{\mathrm{s}}(t) = u_{\mathrm{e}}(t) + \hat{R}i(t)\,, \quad \text{with} \quad \hat{R} = \hat{R}\left(\boldsymbol{z}, i, t\right) = \frac{1}{2}\frac{\mathrm{d}\hat{L}}{\mathrm{d}t}\,, \tag{19}$$

that the static meminductive system can be interpreted as a series interconnection of an energetic meminductive system and a state-dependent resistor $\hat{R}$ representing parametric effects. Like for the energetic definition, $u_{\mathrm{s}}$ denotes the voltage over the static memreactive one-port. Indeed, the resistor can also be interpreted as a memristive system. However, it should be stressed that, in contrast to passive memristive systems, the resistor $\hat{R}$ can particularly be negative. In the proposed special case, $\hat{R}$ can analytically be derived from the meminductive model given in Equation 11

$$\begin{gathered} \hat{R}\left(q\left(t\right), i\left(t\right)\right) = \frac{2\gamma a\left[L_1 - L_0\right]e^{-4\gamma q(t)}}{\left[ae^{-4\gamma q(t)} + 1\right]^2}\, i(t)\,, \quad \text{with} \\ i\left(t\right) \leq 0 \Leftrightarrow \hat{R} \leq 0 \quad \text{and} \quad i\left(t\right) \geq 0 \Leftrightarrow \hat{R} \geq 0\,. \end{gathered} \tag{20}$$

Regarding Equation 20, it is clear that the meminductive system is not necessarily passive for $\hat{L} \geq 0$, moreover, passivity depends on parametric effects represented by the memristive system $\hat{R}$. For example, suitable input current signals or model parameter values could be found in order to operate the meminductive model in an active range, as shown in the simulation result in the following.

*Simulation Results.* Based on the input-output relation described by Equation 18, the hysteresis of a static meminductive system appears in the $\varphi$-$i$-plane. Such a hysteresis is depicted in Fig. 7 (left) for a sinusoidal excitation. The resulting hysteresis curve is comparable to that shown in [15]. Inspecting the stored energy within the device, which is shown in Fig. 7 (right), yields that there is no active behavior with respect to a pure sinusoidal excitation without

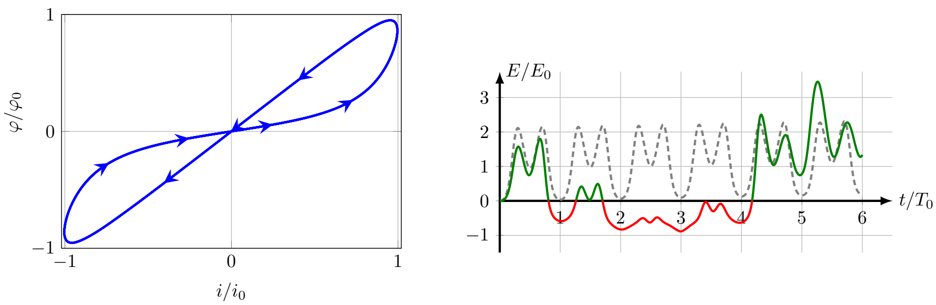


Figure 7: Hysteresis of the meminductive system based on a static definition (left) and the stored energy within the device (right) for an excitation without (gray-dashed) and with offset (colored). Normalization values are $\varphi_0 = 45\ \mu$Wb, $i_0 = 5$ mA, $E_0 = 80$ nJ, and $T_0 = 100$ ms.

an offset (gray-dashed). In spite of that, the stored energy can be negative regarding an excitation with offset, which indicates an active behavior (colored). From a circuit theoretic point of view, a consistent memreactive model should be at least passive independently of particular excitations. Therefore, a modeling approach based on a static definition is not expedient in this context.

*Differential Meminductive System.* A third possibility to model a meminductive system is the use of a differential meminductor

$$u_\mathrm{d}(t) = \hat{L}\left(\boldsymbol{z}, i, t\right) \frac{\mathrm{d}i(t)}{\mathrm{d}t}\ , \tag{21}$$

which reacts to an excitation by a current $i$ with voltage

$$u_\mathrm{d}(t) = \hat{L}\frac{\mathrm{d}i(t)}{\mathrm{d}t} \Leftrightarrow u_\mathrm{d}(t) = u_\mathrm{e}(t) - \hat{R}i(t)\ , \quad \text{with} \quad \hat{R} = \hat{R}\left(\boldsymbol{z}, i, t\right) = \frac{1}{2}\frac{\mathrm{d}\hat{L}}{\mathrm{d}t} \tag{22}$$

Similar to a static meminductive system, such a differential meminductive system is not necessarily passive. Here, the parametric effects are captured in the memristance $-\hat{R}$. It is important to note that a differential meminductor with a negative meminductance can also be (externally) passive.

*Simulation Results.* The input-output relationship of a differential meminductive system defined in Equation 21 yields a hysteresis in the $u$-$\frac{\mathrm{d}i}{\mathrm{d}t}$-plane, cf. Fig. 8 (left). We intended to be comparable with results shown in the common

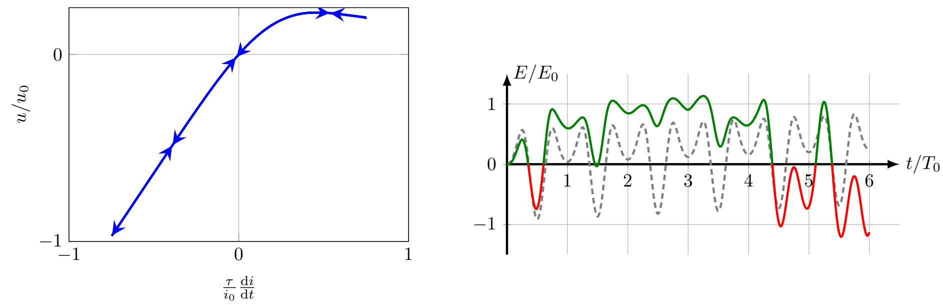


Figure 8: Hysteresis of the meminductive system based on a differential definition (left) and the stored energy within the device (right) for an excitation without (gray-dashed) and with offset (colored). Normalization values are $u_0 = 3,2$ mV, $i_0 = 420$ mA, $\tau = 1$ s, $E_0 = 80$ nJ, and $T_0 = 100$ ms.

literature. Therefore, same parameter values as introduced in [15] has been utilized. But the hysteresis of the differential meminductive system is not obvious for the incorporated parameter values.

On the right-hand side in Fig. 8, the stored energy within the device is shown. It should be emphasized that even for input signals without an offset, the differential meminductive system behaves active (gray-dashed). Therefore, the differential meminductive system is comparable to the static definition and hence it is not a convenient choice considering a consistent modeling approach. Fig. 9 summarizes the three definitions of meminductive systems.

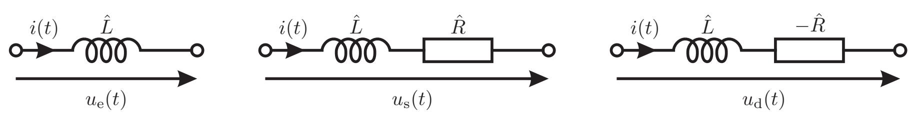


Figure 9: Electrical representation of an energetic (left), static (center) and differential (right) meminductive system

### *3.2. Memcapacitive One-Ports*

From the well-known fact, that a capacitor is dual to an inductor, memcapacitive systems are evidently dual to meminductive systems. Because of the close relation of dual elements, a brief discussion of the modeling of memcapacitive systems is sufficient. As described in the previous section, memcapacitive systems being energetic, static, or differential are considered. Incipient stages of a consistent modeling approach with respect to memcapacitive systems has been already introduced in [25].

### *3.2.1. Energetic Memcapacitive System*

For the modeling of an energetic memcapacitive system the electrical representation in Fig. 10 (middle) is discussed. Here, the current-voltage relationship

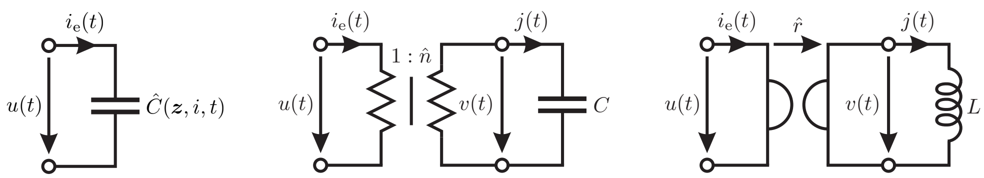


Figure 10: Memcapacitive one-port (left), corresponding electrical representation of a lossless memcapacitive system with a memtransformer (middle), and the dual circuit including a memgyrator (right)

reads

$$i_e(t) = \sqrt{\hat{C}} \frac{\mathrm{d}\sqrt{\hat{C}}u(t)}{\mathrm{d}t}\,, \quad \text{with} \quad \hat{C}\,(\boldsymbol{z}, u, t) = \hat{n}^2\,(\boldsymbol{z}, u, t)\,C\,. \tag{23}$$

Like before and apparent from the given circuit or the power

$$p(t) = u(t) i_e(t) = \frac{\mathrm{d}E(t)}{\mathrm{d}t}\,, \quad \text{with} \quad E(t) = \frac{1}{2}\hat{C}u^2(t) = \frac{1}{2}Cv^2(t) \tag{24}$$

an energetic memcapacitive system is lossless, if and only if $C > 0$ or $L > 0$.

### *3.2.2. Static Memcapacitive System*

A static memcapacitive system is based on a static capacitor, where the current is the time derivative of the charge

$$q(t) = \hat{C}u(t) \Leftrightarrow i_s(t) = i_e + \hat{G}u(t)\,, \quad \text{with} \quad \hat{G} = \hat{G}\,(\boldsymbol{z}, u, t) = \frac{1}{2}\frac{\mathrm{d}\hat{C}}{\mathrm{d}t}\,. \tag{25}$$

Like static meminductive systems, the memductor $\hat{G}$ represents parametric effects of static memcapacitive systems, which in particular can also be negative. As depicted in Fig. 11 (middle), we have a parallel interconnection of an energetic memcapacitor and the conductance $\hat{G}$. Thus, the passivity of a static memcapacitive system depends on $\hat{G}$ and is not ensured for $\hat{C} \geq 0$.

*3.2.3. Differential Memcapacitive System*

A differential memcapacitive system has the current-voltage relation

$$i_{\mathrm{d}}(t) = \hat{C}\frac{\mathrm{d}u(t)}{\mathrm{d}t} \Leftrightarrow i_{\mathrm{d}}(t) = i_{\mathrm{e}}(t) - \hat{G}u(t) \text{ , with } \quad \hat{G} = \hat{G}\left(\boldsymbol{z}, u, t\right) = \frac{1}{2}\frac{\mathrm{d}\hat{C}}{\mathrm{d}t} \text{ , } \quad (26)$$

from which we deduce the parallel interconnection of an energetic memcapacitive system and the memconductance $-\hat{G}$, see Fig. 11 (right). As before, the passivity of a differential memcapacitive system depends on $\hat{G}$. The following Fig. 11 gives an overview of the three modeling approaches of memcapacitive systems.

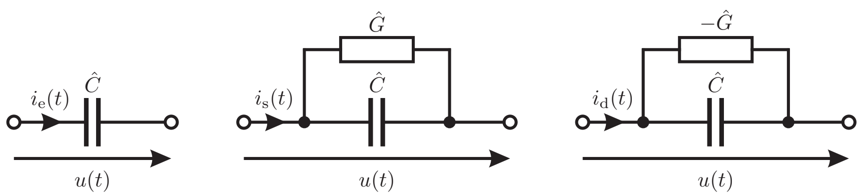


Figure 11: Electrical representation of an energetic (left), static (center) and differential (right) memcapacitive system

If we emphasize the modeling of a lossless memreactive system, it seems intuitive from the argumentations above to model such devices with an energetic inductor or capacitor, respectively. Otherwise, it is hard to ensure passivity without limiting the parameters of the model or the excitation. For example, in [13] a static memcapacitive system has been used and passivity has been confirmed by inspecting the areas of the hysteresis loop in the $q$-$u$-plane. But we must consider, that each hysteresis is only a sample depending on a special excitation. Accordingly, this approach does not provide a criterion for passivity. Due to this fact, the usage of an energetic memreactive system is proposed here. This model has the advantage of being inherently lossless.

## 4. Conclusion

A general approach for modeling lossless memreactive elements has been presented. In this context, an energetic definition of nonlinear reactive elements has been adapted to memreactive models. Investigating the current-voltage relationship of these energetic memreactive models led to energy-neutral two-ports with memory, namely memtransformer and memgyrator. With these novel memory two-ports, an innovative electrical reinterpretation of memory devices has been shown by their corresponding equivalent circuits. It has been turned out, that a suitable choice of complementary constitutive variables plays a key role in the energetically consistent modeling of memreactive elements. With regard to common memreactive models, a paradigm change considering the hysteresis of such elements has been introduced. In this respect, the hysteresis of an energetically defined reactive element with memory should be inspected by the stored energy instead of the charge (memcapacitive) or flux (meminductive). The necessity of an energetically consistent modeling approach has been demonstrated by simulations using a known meminductor model. Regarding a static as well as a differential definition of the meminductive model, it has been shown that for a set of model parameters and a particular excitation, the stored energy within the meminductor can be negative, which indicates an active behavior. In contrast to that, the stored energy within the energetic meminductive model is always positive no matter what parameters and excitations are utilized.

**Acknowledgments**

Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) - 404291403.

**References**

**References**

[1] J. E. Perez-Carmona, H. M. Perez-Meana, J. C. Sanchez-Garcia, Analog adaptive filter LMS algorithm using CMOS 0.5 $\mu$m technology, WSEAS Transactions on Circuits and Systems 7 (9) (2008) 10.

[2] L. O. Chua, Everything You Wish to Know About Memristors But Are Afraid to Ask, Radioengineering 24 (2) (2015) 319–368. doi:10.13164/re.2015.0319.

[3] K. Ochs, E. Hernandez-Guevara, E. Solan, Wave digital emulation of spike-timing dependent plasticity, in: 2017 IEEE 60th International Midwest Symposium on Circuits and Systems (MWSCAS), 2017, pp. 152–155. doi:10.1109/MWSCAS.2017.8052883.

[4] Y. Babacan, F. Kaar, Memristor emulator with spike-timing-dependent-plasticity, AEU - International Journal of Electronics and Communications 73 (2017) 16–22. doi:10.1016/j.aeue.2016.12.025.

[5] J. Zhang, X. Liao, Synchronization and chaos in coupled memristor-based FitzHugh-Nagumo circuits with memristor synapse, AEU - International Journal of Electronics and Communications 75 (2017) 82–90. doi:10.1016/j.aeue.2017.03.003.

[6] Y. Wang, X. Liao, Stability analysis of multimode oscillations in three coupled memristor-based circuits, AEU - International Journal of Electronics and Communications 70 (12) (2016) 1569–1579. doi:10.1016/j.aeue.2016.07.005.

[7] K. Ochs, M. Ziegler, E. Hernandez-Guevara, E. Solan, M. Ignatov, M. Hansen, S. Gill Mahal, H. Kohlstedt, Anticipation of digital patterns, International Journal of Circuit Theory and Applications 46 (2) (2017) 235–243. doi:10.1002/cta.2381.

[8] K. Ochs, E. Hernandez-Guevara, E. Solan, Wave digital information anticipator, in: 2017 IEEE 60th International Midwest Symposium on Circuits and Systems (MWSCAS), 2017, pp. 148–151. doi:10.1109/MWSCAS.2017.8052882.

[9] Y. V. Pershin, M. Di Ventra, Memcapacitive neural networks, Electronics Letters 50 (3) (2014) 141–143. doi:10.1049/el.2013.2463.

[10] A. A. Bessonov, M. N. Kirikova, D. I. Petukhov, M. Allen, T. Ryhnen, M. J. A. Bailey, Layered memristive and memcapacitive switches for printable electronics, Nature Materials 14 (2) (2015) 199–204. doi:10.1038/nmat4135.

[11] P. F. Lee, X. B. Lu, J. Y. Dai, H. L. W. Chan, E. Jelenkovic, K. Y. Tong, Memory effect and retention property of Ge nanocrystal embedded Hf-aluminate high-$k$ gate dielectric, Nanotechnology 17 (5) (2006) 1202–1206. doi:10.1088/0957-4484/17/5/006.

[12] E. Solan, S. Dirkmann, M. Hansen, D. Schroeder, H. Kohlstedt, M. Ziegler, T. Mussenbrock, K. Ochs, An enhanced lumped element electrical model of a double barrier memristive device, Journal of Physics D: Applied Physics 50 (19) (2017) 195102. doi:10.1088/1361-6463/aa69ae.

[13] M. Di Ventra, Y. V. Pershin, L. O. Chua, Circuit Elements With Memory: Memristors, Memcapacitors, and Meminductors, Proceedings of the IEEE 97 (10) (2009) 1717–1724. doi:10.1109/JPROC.2009.2021077.

[14] Y. V. Pershin, M. Di Ventra, Memory effects in complex materials and nanoscale systems, Advances in Physics 60 (2) (2011) 145–227. doi:10.1080/00018732.2010.544961.

[15] D. Biolek, M. Di Ventra, Y. V. Pershin, Reliable SPICE Simulations of Memristors, Memcapacitors and Meminductors, Radioengineering 22 (2013).

[16] J. Martinez-Rincon, M. Di Ventra, Y. V. Pershin, Solid-state memcapacitive system with negative and diverging capacitance, Physical Review B 81 (2010). doi:10.1103/PhysRevB.81.195430.

[17] Y. V. Pershin, M. Di Ventra, Memristive circuits simulate memcapacitors and meminductors, Electronics Letters 46 (7) (2010) 517–518. doi:10.1049/el.2010.2830.

[18] E. Solan, K. Ochs, Parameter identification of a double barrier memristive device, in: 2017 IEEE 60th International Midwest Symposium on Circuits and Systems (MWSCAS), 2017, pp. 851–854. doi:10.1109/MWSCAS.2017.8053057.

[19] K. Ochs, E. Solan, Sensitivity analysis of memristors based on emulation techniques, in: 2016 IEEE 59th International Midwest Symposium on Circuits and Systems (MWSCAS), 2016, pp. 1–4. doi:10.1109/MWSCAS.2016.7869981.

[20] K. Ochs, E. Solan, S. Dirkmann, T. Mussenbrock, Wave digital emulation of a double barrier memristive device, in: 2016 IEEE 59th International Midwest Symposium on Circuits and Systems (MWSCAS), 2016, pp. 1–4. doi:10.1109/MWSCAS.2016.7869946.

[21] K. Ochs, E. Solan, Wave digital emulation of charge-or flux-controlled memristors, in: 2016 IEEE 59th International Midwest Symposium on Circuits and Systems (MWSCAS), 2016, pp. 1–4. doi:10.1109/MWSCAS.2016.7869980.

[22] K. Meerkötter, T. Felderhoff, Simulation of nonlinear transmission lines by wave digital filter principles, in: Proceedings 1992 IEEE International Symposium on Circuits and Systems, Vol. 2, 1992, pp. 875–878 vol.2. doi:10.1109/ISCAS.1992.230082.

[23] K. Ochs, Theorie zeitvarianter linearer Übertragungssysteme, Shaker Verlag, Aachen, 2012.

[24] D. Biolek, Z. Biolek, V. Biolkova, Memristive two-ports, IEEE, 2017, pp. 1–4. doi:10.1109/PATMOS.2017.8106954.

[25] K. Ochs, E. Solan, A consistent modeling of passive memcapacitive systems, in: 2017 IEEE 60th International Midwest Symposium on Circuits and Systems (MWSCAS), 2017, pp. 855–858. doi:10.1109/MWSCAS.2017.8053058.

[26] K. Vrba, F. Kouril, New circuit principles for integrated circuits (2006).