Generalized partial feedback based Orthogonal Space-Time Block Coding

Aydin Sezgin, Gökmen Altay, and Arogyaswami Paulraj

***Abstract*—Recently, a full rate orthogonal space-time block code achieving full diversity while preserving low decoding complexity with very limited feedback was proposed. Based on a feedback of only $p-1$ bits, these codes exhibit a higher coding gain as well. In this work, we propose a low complexity, though close to optimal, feedback selection scheme. Furthermore, we analyze the performance of these codes with arbitrary number of transmit and receive antennas. We show that with this feedback based codes full diversity of $n_T n_R p$ is obtained. Finally, we provide closed form expressions for the ergodic and outage mutual information as well as error rates.**

I. Introduction

There has been considerable work on a variety of schemes which exploit multiple antennas at both the transmitter and receiver in order to obtain transmit and receive diversity and therefore increase the reliability of the system, e.g., orthogonal space-time block codes (OSTBC) [2].

The performance of OSTBC with respect to mutual information has been analyzed (among others) for the uncorrelated Rayleigh fading case in [3], [4]. Unfortunately, the Alamouti scheme [5] for $n_T = 2$ transmit and $n_R = 1$ receive antennas is the only OSTBC, which achieves the capacity due to the rate loss inherent in OSTBC with higher number of transmit antennas [2].

Therefore, based on the assumption that the transmitter has no knowledge about the channel [6], [7] and others designed a quasi-orthogonal space-time block code (QSTBC) with transmission rate one for more than two transmit antennas. Alternatively, it was shown in [8], [9] (and others) that by exploiting limited feedback at the transmitter the performance of the space-time code can be improved significantly. As noticed in [8], limited feedback is provided in an increasing number of standards like e.g. the WCDMA standard. Furthermore, based on a feedback of $p-1$ bits group-coherent codes have been constructed in [8] exhibiting a higher diversity order and coding gain while preserving low decoding complexity. Thus, a space-time code constructed for n_T transmit antennas can easily be extended to $n_T p$ antennas.

In this work, we generalize the work in [8] to an arbitrary number of receive antennas. We show that full diversity of $n_T n_R p$ is achieved with the feedback based OSTBC. Moreover, in [8] an exhaustive search was performed over all possible 2^{p-1} vectors in order to select the optimal $p-1$ feedback bits. In this work, we significantly simplify this feedback selection scheme with a slight performance loss. Furthermore, we analyze the performance of these codes and provide closed form expressions for the ergodic and outage mutual information as well as error rates.

II. System model

As a generalization of the system model in [8], we consider a system with $n_T p$ transmit and n_R receive antennas as shown in Fig. 1. Our system model is defined by

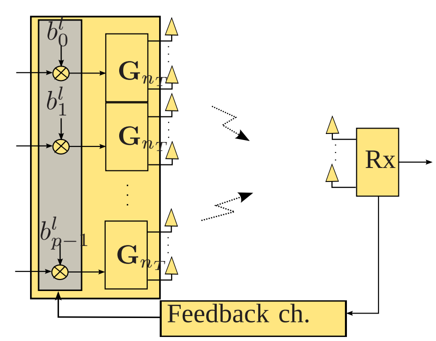

Fig. 1. System Model with feedback channel and transmit sub-blocks.

$$\mathbf{Y} = \mathbf{G}^l_{n_T p}\mathbf{H} + \mathbf{N}\,, \tag{1}$$

where $\mathbf{G}^l_{n_T p}$ is the $(T \times n_T p)$ transmit matrix , $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_{n_R}]$ is the $(T \times n_R)$ receive matrix, $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_{n_R}]$ is the $(n_T p \times n_R)$ channel matrix, and $\mathbf{N} = [\mathbf{n}_1, \ldots, \mathbf{n}_{n_R}]$ is the complex $(T \times n_R)$ white Gaussian noise (AWGN) matrix, where an entry $\{n_{ti}\}$ of $\mathbf{N}$ $(1 \leq i \leq n_R)$ denotes the complex noise at the ith receiver for a given time $t\,(1 \leq t \leq T)$. The real and imaginary parts of n_{ti} are independent and $\mathcal{N}(0,pn_T/(2\mathrm{SNR}))$ distributed, where SNR is the average signal-to-noise ratio ρ. An entry of the channel matrix is denoted by $\{h_{j,i}\}$. This represents the complex gain of the channel between the jth transmit $(1 \leq j \leq n_T p)$ and the ith receive $(1 \leq i \leq n_R)$ antenna, where the real and imaginary parts of the channel gains are independent and normal distributed random variables with $\mathcal{N}$(0,1/2) per dimension. The channel matrix is assumed to be constant for

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a block of T symbols and changes independently from block to block. It is further assumed that the receiver has perfect channel state information (CSI).

A space time block code is defined by its transmit matrix $\mathbf{G}^l_{n_T p}$ with entries $\{x_j\}_{j=1}^r$, which are elements of the vector $\mathbf{x} = [x_1, \ldots, x_r]^T$. The code rate R of a space-time code is defined as $R = r/T$. In this work, the overall transmit matrix $\mathbf{G}^l_{n_T p}$ consist of p sub-blocks (or building elements) $\mathbf{G}_{n_T}$ and is given by (as illustrated in Fig. 1)

$$\mathbf{G}^l_{n_T p} = \left[b^l_0 \mathbf{G}_{n_T}, b^l_1 \mathbf{G}_{n_T}, b_2 \mathbf{G}_{n_T}, \ldots, b^l_{p-1} \mathbf{G}_{n_T}\right] \quad (2)$$

with $l = 1, 2, \ldots, p$, assuming that the feedback b^l_k, $1 \le k \le p-1$, ($b^l_0 = 1$ is preset) is binary and given as $b_k = \{-1, 1\}$. The matrix $\mathbf{B}$ given by

$$\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_l, \ldots, \mathbf{b}_p] \quad (3)$$

contains a subset of all possible (2^{p-1}) feedback vectors $\mathbf{b}_l$ of length p. $\mathbf{B}$ can be constructed similar to Hadamard matrices [10]. We will show later on that considering this specific set of p vectors out of all 2^{p-1} vectors is sufficient to achieve a near-optimal performance. Similarly to Hadamard matrices, $\mathbf{B}$ has some nice properties such as

$$\sum_{k=0}^{p-1} b^l_k b^{l'}_k = \sum_{l=1}^{p} b^l_k b^l_{k'} = 0 \quad (4)$$

$$\sum_{k=0}^{p-1} b^l_k b^l_k = \sum_{l=1}^{p} b^l_k b^l_k = p \quad (5)$$

which will be utilized later on. Note that Hadamard-type matrices do not exist for all dimensions. The performance expressions derived below are derived for the cases with the dimension of $\mathbf{B}$ given as a power of 2, i.e. when Hadamard-type matrices exist. However, full diversity is achieved for all cases, i.e. $p = 2, 3, \ldots$, which was already shown in [8]. In addition to this, the expression which is derived below for the outage probability can still be used as a lower bound, while the upper bound for the bit error rate and the lower bound for the average rate serve then as approximations.

Depending on the given performance criteria discussed later on, the receiver selects the respective feedback vector, which is then conveyed over a control or feedback channel to the transmitter. Note that we use OSTBC as building elements $\mathbf{G}_{n_T}$. However, note that there exist a tradeoff between code rate of the OSTBC and the number of feedback bits depending on the selected space-time code.

III. Receiver Processing

Each vector $\mathbf{h}_i$ of the channel in (1) contains $n_T p$ coefficients and may be expressed in a stacked form given by $\mathbf{h}_i = [(\mathbf{h}^1_i)^T, \ldots, (\mathbf{h}^k_i)^T, \ldots, (\mathbf{h}^p_i)^T]^T$, where each subset $\mathbf{h}^k_i$ is associated with the corresponding subblock of $\mathbf{G}^l_{n_T p}$. At each receive antenna i, the received signal can be rewritten as (assuming $b^l_0 = 1$ for all l)

$$\begin{aligned}\mathbf{y}^l_i &= \mathbf{G}_{n_T}\mathbf{h}^1_i + b^l_1 \mathbf{G}_{n_T}\mathbf{h}^2_i + \cdots + b^l_{p-1}\mathbf{G}_{n_T}\mathbf{h}^p_i + \mathbf{n}_i \\ &= \mathbf{G}_{n_T}\left(\mathbf{h}^1_i + b^l_1 \mathbf{h}^2_i + \cdots + b^l_{p-1}\mathbf{h}^p_i\right) + \mathbf{n}_i = \mathbf{G}_{n_T}\hat{\mathbf{h}}^l_i + \mathbf{n}_i\end{aligned}$$

Thus, the entries of $\hat{\mathbf{h}}^l_i$ are obtained in the following way

$$\hat{h}^l_{j,i} = \mathbf{b}^T_l [h_{j,i}, h_{n_T+j,i}, \ldots, h_{(p-1)n_T+j,i}]^T .$$

After some manipulations (particularly complex-conjugating) the received signal at each receive antenna can be rewritten as

$$\mathbf{y}'_i = \mathbf{R}^l_i \mathbf{x} + \mathbf{n}'_i ,$$

where $\mathbf{R}^l_i = [\mathbf{H}'_{i1} + b^l_1 \mathbf{H}'_{i2} + b^l_2 \mathbf{H}'_{i3} + \cdots + b^l_{p-1}\mathbf{H}'_{ip}]$ and $\mathbf{H}'_{ik}$ is the equivalent channel matrix of the OSTBC containing channel coefficients of the subset $\mathbf{h}^k_i$ given by

$$\mathbf{h}^k_i = [h_{(k-1)n_T+1,i}, \ldots, h_{kn_T,i}]^T$$

and their conjugates [3]. The complete receive vector is then accordingly given by

$$\mathbf{y}' = \mathbf{R}^l \mathbf{x} + \mathbf{n}' , \quad (6)$$

where $\mathbf{y}' = [(\mathbf{y}'_1)^T, \ldots, (\mathbf{y}'_{n_R})^T]^T$, $\mathbf{R}^l = [(\mathbf{R}^l_1)^T, \ldots, (\mathbf{R}^l_{n_R})^T]^T$, and $\mathbf{n}' = [(\mathbf{n}'_1)^T, \ldots, (\mathbf{n}'_{n_R})^T]^T$. An illustrative example of the derivation above is given in [8].

In the remainder of the paper, we will consider the Alamouti code as the building element. The results can be easily extended to other OSTBC in a similar way. In the following we construct a generalized expression for the equivalent channel matrix for the ith receive antenna. First of all, due to the linearity of the code [3] we are able to write the equivalent channel matrix in the following form

$$\mathbf{H}'_{ik} = \sum_{m=1}^{r=2} \mathbf{C}_m h_{2k+m,i} + \mathbf{D}_m h^*_{2k+m,i},$$

where the matrices $\mathbf{C}_m$, $\mathbf{D}_m$ completely specify the code [3]. Thus, $\mathbf{R}^l_i$ can be written as follows

$$\mathbf{R}^l_i = \sum_{k=0}^{p-1} b^l_k \left(\sum_{m=1}^{r=2} \mathbf{C}_m h_{2k+m,i} + \mathbf{D}_m h^*_{2k+m,i}\right).$$

After channel matched filtering to (6) and using the properties of $\mathbf{C}_m$ and $\mathbf{D}_m$ for the Alamouti code [3], we have

$$\begin{aligned}(\mathbf{R}^l_i)^H(\mathbf{R}^l_i) &= \mathbf{C}_1 \underbrace{\left(\sum_{k=0}^{p-1} b^l_k h^*_{2k+1,i}\right)\left(\sum_{k=0}^{p-1} b^l_k h_{2k+1,i}\right)}_{\alpha^i_l} \\ &+ \mathbf{C}_1 \underbrace{\left(\sum_{k=0}^{p-1} b^l_k h_{2k+2,i}\right)\left(\sum_{k=0}^{p-1} b^l_k h^*_{2k+2,i}\right)}_{\epsilon^i_l} - \mathbf{D}_1 \alpha^i_l - \mathbf{D}_1 \epsilon^i_l \\ &= (\mathbf{C}_1 - \mathbf{D}_1)(\alpha^i_l + \epsilon^i_l) = \gamma^i_l \mathbf{I}.\end{aligned}$$

for each receive antenna i. Interestingly, γ^i_l can be represented in the following quadratic form

$$\gamma^i_l = \sqrt{p}\mathbf{h}^H_i \mathbf{M}_l \mathbf{h}_i \sqrt{p}$$

where

$$\mathbf{M}_l = \frac{1}{p}\left(\mathbf{b}_l \otimes [1\ 0]^T\right)\left(\mathbf{b}_l \otimes [1\ 0]^T\right)^T , \quad (7)$$

with $1 \leq l \leq p$, where $\otimes$ denotes the Kronecker product. Taking into account all receive antennas results in

$$\gamma_l = \sum_{i=1}^{n_R} \sqrt{p}\mathbf{h}_i^H \mathbf{M}_l \mathbf{h}_i \sqrt{p}. \tag{8}$$

Thus, for a specific feedback vector, the equivalent channel gain is completely specified by γ_l. Since we are feeding back the feedback vector which provides the highest channel gain, the resulting equivalent channel gain is given by $\max(\gamma_1, \ldots, \gamma_p)$. A better performance can be achieved by considering all possible feedback vectors as was done in [8], resulting in equivalent channel gains $(\gamma_1, \ldots, \gamma_{2^{p-1}})$. However, as we will see later on, the additional gains achieved with this strategy are minor especially considering the exponential complexity due to exhaustive search.

Similar to the OSTBC, the feedback OSTBC are also diagonalizing the channel, such that simple decoding can be applied. An immediate question is whether full diversity can be achieved with the feedback OSTBC as well. In order to to show this, we prove the following lemmata in order to completely characterize the statistics of the γ_l.

Lemma 3.1: The matrices $\mathbf{M}_l$ given in (7), $1 \leq l \leq p$, are mutually orthogonal matrices.

Proof: Orthogonality is given if $(\mathbf{M}_l)^H \mathbf{M}_{l'} = \mathbf{0}$, $l \neq l'$ holds. It turns out that the entries in $(\mathbf{M}_l)^H \mathbf{M}_{l'}$, $l \neq l'$ are given by

$$\frac{1}{p^2} b_k^l b_{k'}^{l'} \left(\sum_{k=1}^{p-1} b_k^l b_k^{l'} \right). \tag{9}$$

From (4) we know that $\sum_{k=0}^{p-1} b_k^l b_k^{l'} = 0$ holds. Thus, (9) is zero and orthogonality is given. ■

Lemma 3.2: The $\mathbf{M}_l$ given in (7), $1 \leq l \leq p$, are idempotent matrices.

Proof: To prove this we need to show that $(\mathbf{M}_l)^H \mathbf{M}_l$=$\mathbf{M}_l$. Note that the entry of $(\mathbf{M}_l)^H \mathbf{M}_l$ in column m and row m' are given by

$$\frac{1}{p^2} b_m^l b_{m'}^l \left(\sum_{m=0}^{p-1} b_m^l b_m^l \right) \overset{(5)}{=} \frac{1}{p} b_m^l b_{m'}^l,$$

which equals the (m, m')th entry of $\mathbf{M}_l$, $1 \leq l \leq p$. ■

Lemma 3.3: The rank of the matrices $\mathbf{M}_l$ given in (7), $1 \leq l \leq p$, equals $n_T = 2$.

Proof: The proof can be shown in two steps. First, by performing row and column permutations, the $\mathbf{M}_l$ can be rewritten as a block diagonal matrix, where the even columns and rows appear in the first upper block and the odd columns and rows appear in the lower block. As a second step, each permuted $\mathbf{M}_l$ can be rewritten such that the rank is given by

$$\mathrm{rank}\,(\mathbf{M}_l) = \frac{1}{p} \mathrm{rank} \left(\mathbf{D}^l \begin{bmatrix} \mathbf{A}^l & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^l \end{bmatrix} \right) \tag{10}$$

where $\mathbf{D}^l$ is a full rank diagonal matrix given by $\mathbf{D}^l = \mathrm{diag}(\mathbf{b}_l^T, \mathbf{b}_l^T)$, $\mathbf{A}^l$ is a rank one matrix. Every row of $\mathbf{A}^l$ is equal to $(\mathbf{b}^l)^T$. Thus, the overall matrix $\mathbf{M}_l$ has rank two. ■

Lemma 3.4: Suppose that the $\mathbf{M}_l$ are given as in (7). It holds that

$$\sum_{l=1}^{p} \mathbf{M}_l = \mathbf{I}. \tag{11}$$

Proof: The sum of the diagonal elements of the matrices $\mathbf{M}_l$ are given by

$$\frac{1}{p} \sum_{l=1}^{p} b_k^l b_k^l \overset{(5)}{=} 1. \tag{12}$$

Similarly, the non-diagonal elements of the summed matrix are given by

$$\sum_{\substack{l=1, \\ k \neq k'}}^{p-1} b_k^l b_{k'}^l \overset{(4)}{=} 0, \tag{13}$$

which is nothing else than the product of two different rows of the Hadamard matrix. ■

Theorem 3.1 (Theorem 5.1.6 in [11]): Let $\mathbf{x} \sim \mathcal{N}_{pn_T}(\mathbf{0}, \mathbf{I})$ and $\mathbf{x}^H \mathbf{x} = \mathbf{x}^H \mathbf{A}_1 \mathbf{x} + \mathbf{x}^H \mathbf{A}_2 \mathbf{x}$, $\mathbf{A}_1 = \mathbf{A}_1^H$, $\mathbf{A}_2 = \mathbf{A}_2^H$, where $\mathbf{A}_1$ is idempotent of rank $r < pn_T$. Then $\mathbf{x}^H \mathbf{A}_1 \mathbf{x} \sim \chi_r^2$, $\mathbf{x}^H \mathbf{A}_2 \mathbf{x} \sim \chi_{pn_T - r}^2$ and $\mathbf{x}^H \mathbf{A}_1 \mathbf{x}$ and $\mathbf{x}^H \mathbf{A}_2 \mathbf{x}$ are independently distributed.

Remark 3.1: The γ_l have also the same quadratic structure as the variables in Theorem 3.1, with the $\mathbf{M}_l$ fulfilling the necessary criteria mentioned in the above theorem. Thus, the γ_l are independently $\chi_{4n_R}^2$ distributed.

Based on the previous lemmata, we state the following theorem

Theorem 3.2: The feedback OSTBC in (2) achieve full diversity.

Proof: The channel gain of the feedback OSTBC can be lower and upper bounded by

$$\bar{\gamma} = \frac{1}{p} ||\mathbf{H}||_F^2 = \frac{1}{p} \sum_{=1}^{p} \gamma_l \leq \max(\gamma_1, \ldots, \gamma_p) \leq \sum_{=1}^{p} \gamma_l = ||\mathbf{H}||_F^2. \tag{14}$$

Thus, the channel gain is limited by two expressions, which both provide full diversity of $2pn_R n_T$. ■

The advantage of the above derivations is that we have exactly specified the resulting equivalent channel gain. For comparison, in [8] only an approximation was given.

We have the following probability density function (pdf)

$$f_{\Gamma_l}(\gamma_l) = \frac{\gamma_l^{2n_R - 1} e^{-\gamma_l}}{(2n_R - 1)!} ,$$

and the following cumulative distribution function (cdf)

$$F_{\Gamma_l}(\gamma_l) = 1 - \sum_{j=0}^{2n_R - 1} \frac{\gamma_l^j e^{-\gamma_l}}{j!} ,$$

for the individual γ_l. Thus, the feedback bit is used to select $z = \max(\gamma_1, \gamma_2, \ldots, \gamma_p)$ where the cdf and pdf of z are given by

$$F(z) = F_{\Gamma_l}^p(\gamma_l) \text{ and } p_Z(z) = pF(z)f(z) , \tag{15}$$

respectively, in order to achieve full diversity or high capacity.

In the following section, the performance of the proposed scheme with respect to mutual information and error rates is analyzed. We use the same SNR based approach as in [12], [13]

IV. Performance Analysis

The mutual information achievable with the proposed scheme is given by

$$I_{\mathrm{OF}} = R\log_2\left(1+\frac{2\rho}{pn_T}\max(\gamma_1,\gamma_2,\ldots,\gamma_p)\right) . \quad (16)$$

A. Outage probability

The outage probability P_{out} achievable with the proposed scheme is defined as the probability that I_{OF} is smaller than a certain transmission rate R_T, i.e.

$$P_{out}(R_T,n_T,n_R,\rho) = \Pr[I_{\mathrm{OF}} < R_T]$$
$$= \Pr\left[\max(\gamma_1,\gamma_2,\ldots,\gamma_p) < (2^{R_T/R}-1)\frac{pn_T}{2\rho}\right] ,$$

where R_T is the transmission rate. After some manipulations, we arrive at

$$P_{out}(R_T,n_T,n_R,\rho) = \left(1-\frac{\Gamma\left(n_Tn_R,(2^{R_T/R}-1)\frac{pn_T}{2\rho}\right)}{\Gamma(n_Tn_R)}\right)^p ,$$

where $\Gamma(\cdot,\cdot)$ and $\Gamma(\cdot)$ are the incomplete and the complete Gamma function [14, p.940,8.350(2)], respectively.

In Fig. 2, the outage probability of the proposed scheme with $pn_T = 4$ transmit and $n_R = 1$ receive antennas is depicted. In addition to this, the outage probability of a idealistic rate one non-feedback OSTBC for $pn_T = 4$ is depicted. From the figure, we observe that the proposed curve achieves full diversity. Furthermore, a coding gain of about 1 dB in comparison to the idealistic rate one OSTBC is achieved.

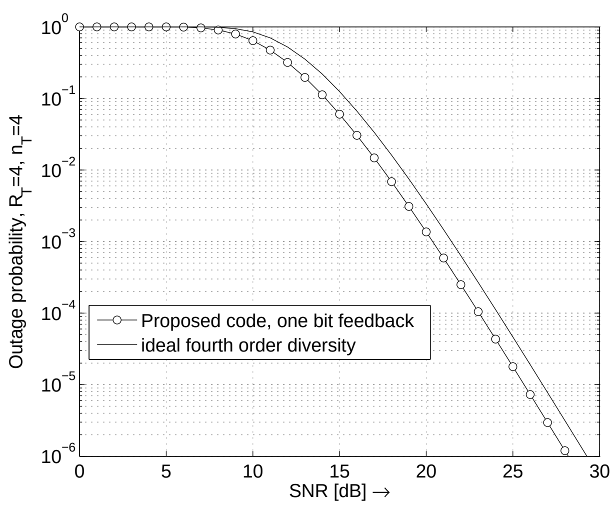

Fig. 2. Outage probability of the proposed scheme with $n_R = 1$ receive and $n_Tp = 4$ transmit antennas, transmission rate $R_T = 4$.

B. Average mutual information

In order to analyze the average mutual information, we use the inequality in (14) in order to obtain a lower bound on $I_{\mathbf{OF}}$ in (16) given by

$$I_{\mathrm{OF}}^{lb} = R\log_2\left(1+\frac{2\rho}{pn_T}\bar{\gamma}\right) \le I_{\mathrm{OF}} . \quad (17)$$

Recall that $\bar{\gamma}$ in (14) is also chi-square distributed, but with $2pn_Rn_T$ degrees of freedom. Averaging over all channel realizations results in (by using integration by parts)

$$\mathbb{E}[I_{\mathrm{OF}}] \ge \mathbb{E}[I_{\mathrm{OF}}^{lb}] = \frac{1}{\ln(2)}\sum_{k=0}^{pn_Rn_T-1}\left(\frac{pn_T}{2\rho}\right)^{pn_Rn_T-k-1} \times e^{\frac{pn_T}{2\rho}}\Gamma\left(1-(pn_Rn_T-k),\frac{pn_T}{2\rho}\right) . \quad (18)$$

In Fig. 3, the average mutual information of the proposed scheme with $n_T = 4$ transmit and $n_R = 1$ and also with $n_R = 2$ receive antennas is depicted. In addition to this, the lower bound given by (18) is depicted. From the figure, we observe that the difference between the lower bound and the actual average mutual information is only about 1 dB for the case of $n_R = 1$ receive antenna. Furthermore, the gap is slightly reduced in case of $n_R = 2$ receive antennas. The behavior of space-time transmission schemes with respect

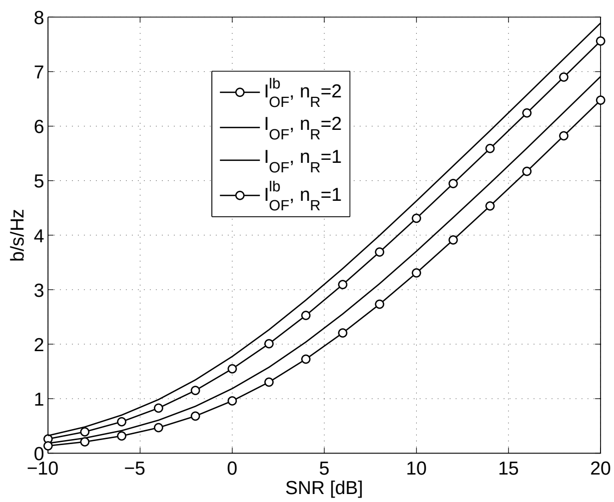

Fig. 3. Average mutual information of the proposed scheme and lower bound with $n_R = 1$ and $n_R = 2$ receive antennas, respectively, and $n_Tp = 4$ transmit antennas.

to mutual information is an important benchmark from an information theoretic point of view. What is also interesting for the practical system designer, however, is the error rate performance, which is analyzed in the following section.

C. Error rates

Assuming QPSK modulation and maximum likelihood (ML)-decoding, the conditional BER for the proposed scheme is given by

$$P_e = Q\left(\sqrt{2R\frac{2\rho}{pn_T}\max(\gamma_1,\gamma_2\ldots,\gamma_p)}\right) .$$

By using (14), the conditional BER may be upper bounded as follows

$$P_e \le Q\left(\sqrt{\frac{4R\rho}{pn_T}\bar{\gamma}}\right) = P_e^{ub} .$$

Using standard techniques [15], the average BER is obtained by averaging the conditional BER over all channel realizations and is given by

$$P_e^{ub} = \left(\frac{1}{2}(1-\mu)\right)^{pn_Tn_R} \sum_{k=0}^{pn_Tn_R-1} \binom{pn_Tn_R-1+k}{k} \left(\frac{1}{2}(1+\mu)^k\right), \quad (19)$$

where

$$\mu = \sqrt{\frac{\rho}{\frac{pn_T}{2}+\rho}}.$$

In Fig. 4, the bit error rate performance and the upper bound in (19) of the proposed scheme with $n_Tp = 4$ transmit and $n_R = 1$ receive antennas is depicted. In addition to this, the bit error rate performance of an OSTBC for four transmit antennas and rate $R = 3/4$ [16] is depicted. From the figure we observe that the upper bound results only in a horizontal shift compared to the exact error rate performance of the proposed scheme. Furthermore, a coding gain of about 2.5 dB in comparison to the rate $R = 3/4$ OSTBC [16] is achieved. In Fig. 5, the bit error rate performance of the proposed scheme with $n_Tp = 8$ transmit and $n_R = 1$ receive antennas is depicted. In addition to this, the bit error rate performance by using the exhaustive search over all possible feedback vector [8] is depicted. From the figure we observe that the exhaustive search results only in a horizontal shift of less than 1 dB compared to the performance of the proposed scheme.

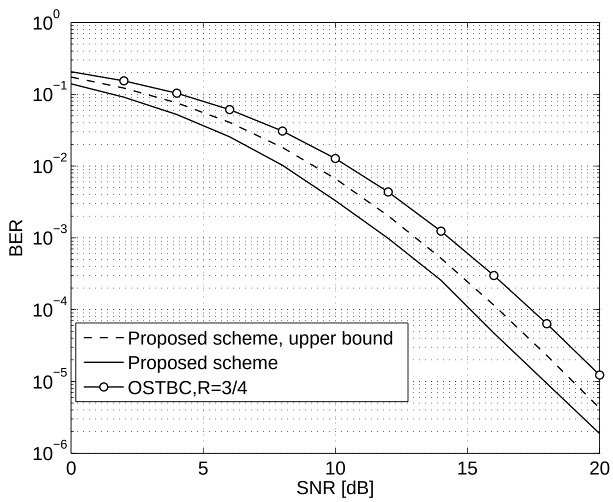

Fig. 4. Error rate performance of the proposed scheme with $n_R = 1$ receive and $n_Tp = 4$ transmit antennas and QPSK.

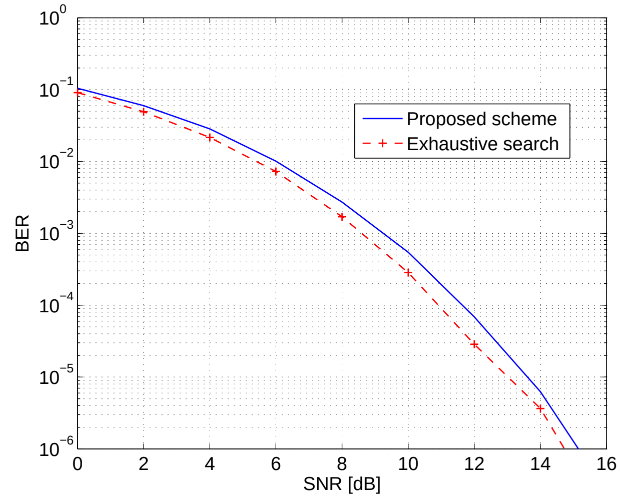

Fig. 5. Error rate performance using the exhaustive search proposed in [8] and the proposed scheme with $n_R = 1$ receive and $n_Tp = 8$ transmit antennas and QPSK.

V. Acknowledgement

We would like to thank the first reviewer for the detailed and insightful comments, which significantly enhanced the quality and readability of the paper.

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