

# Tile-based MIMO OFDMA systems: The Impact of Inaccurate Channel State Information

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**Abstract**—Advanced multiple-input multiple-output (MIMO) techniques frequently entail high computational complexity and large feedback overhead, which often prevents their full realization in practice. In order to alleviate this problem, resource bundling has been proposed. By applying the same precoding on a block (tile) of adjacent OFDM subcarriers, both the complexity of the system and the feedback overhead can be reduced significantly, at the cost of a slight performance degradation.

In this paper, we consider the realistic situation where the available channel state information is noisy or inaccurate due to estimation errors or feedback delays. The robustness of the performance of a multi-user uplink system is studied for various tile sizes. We show that under channel uncertainty, optimal performance is no longer achieved with the smallest possible tile size. Thus, in this setting, tile-based processing achieves a performance gain, in addition to reducing complexity and overhead.

## I. INTRODUCTION

Recent advances in the analysis of theoretical performance limits show that significant gains are achievable in single-carrier and multi-carrier communication systems using multiple-antenna techniques [1]. To achieve these gains, very sophisticated and sometimes computationally complex transmit and receive strategies have to be applied which adapt nonlinearly to current channel state information (CSI). [2].

Indeed, the exploitation of CSI at the transmitter is an ongoing research topic [3], [4]. It has been shown that CSI at the transmitter can significantly improve the performance and reliability of multiple antenna multi-user systems [5], [6], [7]. For instance, transmit strategies for the cyclic prefix orthogonal frequency division multiplex (CP-OFDM) multiple-input multiple-output (MIMO) multiple access channel (MAC) and broadcast channel (BC) were proposed, which optimize the throughput, fairness, or delay performance of the system [8], [9]. Under perfect [10] and long-term CSI [11], [12], the optimal linear precoding matrices were found using a centralized approach. In order to apply the linear precoding at the mobile stations (MS) in the uplink, the precoding matrices are

obtained centrally at the base station (BS), and then fed back to the mobiles via a control channel.

However, the realization and implementation of such techniques is often limited by the capability and impairments of existing hardware, as well as other practical issues such as mobility [13], feedback, imperfect channel knowledge and synchronization. Thus, the reduction of feedback and complexity, as well as the robustness to channel knowledge inaccuracies are major issues for future communication systems.

The approach for feedback and complexity reduction in this paper is to bundle available resources into groups, and apply the same signal processing within each group. Specifically, we aggregate a subset of adjacent OFDM subcarriers in the time and frequency domains into a tile (sometimes called time-frequency chunk, or subchannel), and use a common precoder/decoder matrix for the entire tile. This subchanneling strategy is already incorporated in current wireless standards like IEEE 802.16 (WiMAX) and IEEE 802.20 [14]. There, the complete time-frequency grid is subdivided into tiles [15], [16], [17].

Our focus in this work is on bundling in the frequency domain, i.e., we focus on one-dimensional tiles. However, the generalization to the two-dimensional case (time and frequency) or even the three-dimensional case (i.e. antennas) is straightforward. As stated earlier, for all subcarriers within a tile, the same precoding strategy is applied, reducing the complexity and the overhead considerably. In addition, we assume that the available channel state information is inaccurate (either through estimation errors or outdatedness from feedback delays), and study the robustness of tiling strategies under a sum-rate objective. The results are illustrated by numerical simulations.

## II. SYSTEM MODEL

**MIMO multi-user uplink.** Consider an ideal multi-user OFDMA uplink system with  $K$  users,  $N$  OFDM subcarriers,  $n_T$  transmit antennas at each mobile station, and  $n_R$  receive antennas at the base station. The received signal on subcarrier  $n$ , for  $1 \leq n \leq N$ , is given by

$$\mathbf{y}_n = \sum_{k=1}^K \mathbf{H}_{n,k} \mathbf{x}_{n,k} + \mathbf{n}_n. \quad (1)$$

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Here,  $k$  is the user index, for  $1 \leq k \leq K$ . The matrices  $\mathbf{H}_{n,k}$  are the fading channel gains, whose entries are modeled as i.i.d. zero-mean circularly symmetric complex Gaussian distributed with unit variance. Furthermore,  $\mathbf{x}_{n,k}$  is the transmit vector, and  $\mathbf{n}_n$  is white Gaussian noise with variance  $\sigma_n^2 = \frac{1}{\rho}$ .

Let us define the transmit covariance matrices

$$\mathbf{Q}_{n,k} = \mathbb{E}[\mathbf{x}_{n,k}\mathbf{x}_{n,k}^H]. \quad (2)$$

Each user is constrained by an individual power budget  $P_k$ , such that  $\sum_{n=1}^N \text{tr}(\mathbf{Q}_{n,k}) \leq P_k$ .

**Centralized processing and channel inaccuracies.** Based on estimates of channel matrices for all users and subchannels,  $\{\hat{\mathbf{H}}_{k,n}, \forall k, n\}$ , the base station centrally computes the transmit covariance matrices to be used, which are then fed back to the mobile stations using a feedback link. The users obey the prescribed covariance matrices through linear precoding.

However, the channel estimates  $\hat{\mathbf{H}}_{k,n}$  on which the computation is based may deviate from the true channel state at the time of transmission. This is mainly due to two effects: 1) the channel estimation at the base station is noisy to begin with, and 2) the true channel has evolved during the time needed for computation and feedback. We model this inaccuracy as

$$\hat{\mathbf{H}}_{n,k} = \sqrt{1-\alpha}\mathbf{H}_{n,k} + \sqrt{\alpha}\mathbf{W}_{n,k}. \quad (3)$$

The entries of the matrix  $\mathbf{W}_{k,n}$  are i.i.d. zero-mean complex Gaussian of unit variance, independent of all  $\mathbf{H}_{n,k}$ . The weight factor  $\alpha$ , with  $0 < \alpha < 1$ , parameterizes the degree of inaccuracy. If  $\alpha = 0$ , the channel estimate matches the true channel perfectly. On the other extreme,  $\alpha = 1$  means that the channel estimate is independent of the true channel, and thus essentially useless.

**Tile-based processing.** To reduce the amount of feedback needed for the scheme, we may use the *same* transmit covariance matrix for several neighboring subcarriers. For simplicity, we will assume rectangular regions in the time-frequency plane. All subcarriers in the region will use the same processing (same  $\mathbf{Q}_{n,k}$ ), and we thus term the region a tile (see Figure 1). The tile size determines the amount of feedback reduction. In the multi-user case, the tile size also defines the granularity of resource allocation.

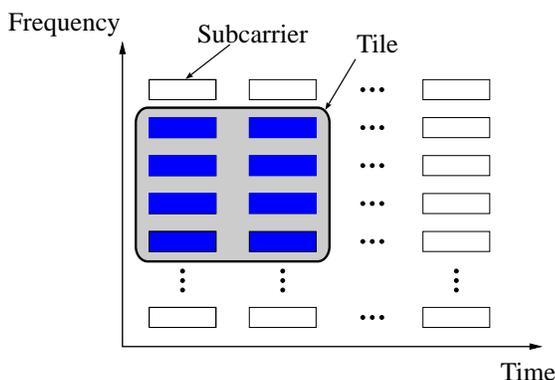


Fig. 1. Tile Processing: One tile contains several adjacent frequency subcarriers and OFDM symbols

Tile processing reduces the required feedback by a factor equal to the tile size, i.e., the total number of subcarriers in the tile. If the channel state information is accurate, this

reduction of feedback overhead, however, comes at a cost of performance (with high probability, no common covariance matrix exists that is locally optimal for each subcarrier in a tile). In other words, the optimal tile size from a strict performance viewpoint is a single subcarrier.

In this paper, we study the effect of inaccurate channel state information. In this situation, as we will see, the smallest possible tile size is no longer optimal. Instead, there is a performance-optimal tile size. Thus, under channel uncertainty, using tile processing helps both reduce the overhead and improve performance.

### III. SINGLE-USER CASE

#### A. Optimal tile-based processing

The problem statement of tile processing is as follows. We assume that each tile comprises  $B$  subcarriers and is assigned to one user. The total of  $N$  subcarriers is thus divided into  $N/B$  tiles<sup>1</sup>. In the following, instead of indexing the subcarriers by their number  $n = 1, \dots, N$ , we will instead use two indices  $(m, b)$ , where  $m = 1, \dots, N/B$  is the tile number, and  $b = 1, \dots, B$  is the subcarrier number within the tile. Furthermore, we will drop the user index  $k$  in this section, since we consider the single-user case. For example, instead of writing  $\mathbf{H}_{n,k}$ , we will use  $\mathbf{H}_{m,b}$ , where  $n = (m-1)B + b$  and  $k = 1$ .

Similar to [18], the base station computes the precoding matrices by treating the available channel information  $\hat{\mathbf{H}}_{m,b}, \forall m, b$  as if it was accurate, a method that may be called a *naive* precoder. Then we maximize the overall average sum rate as in

$$\begin{aligned} & \text{maximize} \quad \sum_{m=1}^{N/B} \sum_{b=1}^B \mathbb{E} \left[ \log \left| \mathbf{I} + \rho \hat{\mathbf{H}}_{m,b} \mathbf{Q}_m \hat{\mathbf{H}}_{m,b}^H \right| \right], \quad (4) \\ & \text{subject to} \quad \mathbf{Q}_m \succeq \mathbf{0} \\ & \quad \quad \quad \sum_m \text{tr}(\mathbf{Q}_m) \leq \frac{P}{B}, \end{aligned}$$

where the optimization variables are the  $\mathbf{Q}_m$  for  $m = 1, \dots, N/B$ , i.e., the per-tile transmit covariance matrices. The problem data are  $N, B, P, \rho$ , and  $\hat{\mathbf{H}}_{m,b}$  for  $m = 1, \dots, N/B, b = 1, \dots, B$ , where  $P$  is the total available power,  $\rho = 1/\sigma_n^2$  is the inverse noise variance, and  $\hat{\mathbf{H}}_{m,b}$  is the available (outdated) channel information in the  $b$ th subcarrier of the  $m$ th tile. Naturally, the same precoding matrix  $\mathbf{Q}_m$  is used for all subcarriers within a tile and thus in (4) the index  $b$  is unnecessary.

In the  $m$ th tile, the problem is thus to find a common spatial MIMO transmit signature that will be applied to all subcarriers in the tile, where in general the channel is frequency-selective, i.e.,  $\mathbf{H}_{m,b} \neq \mathbf{H}_{m,b'}$  for  $b \neq b'$ . The tile-wise problems are coupled across tiles only via the common power constraint.

Disregarding the coupling between tiles for the moment, the optimization in the  $m$ th tile can be solved by using a fixed-

<sup>1</sup>We assume that  $B$  is an integer divisor of  $N$ .

point algorithm [17] with update rule

$$\mathbf{Q}_m^{\ell+1} = \frac{\mathbb{E} \left[ P \sum_{b=1}^B \left( \mathbf{Q}_m^{\ell,1/2} \mathbf{H}_{m,b}^H \mathbf{G}_{m,b} \mathbf{H}_{m,b} \mathbf{Q}_m^{\ell,1/2} \right) \right]}{\mathbb{E} \left[ \text{tr} \sum_{b=1}^B \left( \mathbf{Q}_m^{\ell,1/2} \mathbf{H}_{m,b}^H \mathbf{G}_{m,b} \mathbf{H}_{m,b} \mathbf{Q}_m^{\ell,1/2} \right) \right]}$$

where

$$\mathbf{G}_{m,b} = [\mathbf{I} + \rho \mathbf{H}_{m,b} \mathbf{Q}_m^{\ell} \mathbf{H}_{m,b}^H]^{-1}$$

with initialization  $\mathbf{Q}_m^0 = \frac{P}{n_T} \mathbf{I}$ . The algorithm was shown to converge to the optimal precoding matrix [17].

To incorporate the coupling between the chunks, the tilewise solutions are combined via a spectral power allocation across the tiles. In other words, the transmit power of the user is distributed across the  $N/B$  tiles to maximize capacity. The spectral power allocation takes the form of simple water-filling [19] over the tiles since they are orthogonal due to ideal CP-OFDM. As expected for waterfilling, at low SNR the power is only allocated to the strongest tile, while at high SNR the power is distributed equally across the tiles.

### B. Channel inaccuracy and tile size

In this section, a lower bound for the achievable rate as a function of the subchannel size is derived in order to shed some light on the interplay between channel inaccuracy and tile size.

Let  $R^*$  and  $\{\mathbf{Q}_m^*\}$  denote the optimal value and optimal point of (4), respectively. Obviously, if the  $\{\mathbf{Q}_m^*\}$  are plugged into the objective function, the optimal rate value  $R^*$  is obtained. On the other hand, using any other assignment  $\{\mathbf{Q}_m\}$  will yield a lower bound on  $R^*$ , as long as that assignment satisfies the constraints of (4).

In particular, for all  $m$ , let us replace  $\mathbf{Q}_m^*$  with the weighted sum

$$\mathbf{Q}_m = \sum_{b'=1}^B \alpha_{m,b'} \mathbf{Q}_{m,b'}, \quad (5)$$

where  $\alpha_{m,b'} \geq 0$ ,  $\sum_{b'=1}^B \alpha_{m,b'} = 1$  are coefficients to be chosen later. Here,  $\mathbf{Q}_{m,b'}$  is the optimal covariance matrix for the (true) channel  $\mathbf{H}_{m,b'}$ , assuming equal power distribution among all subcarriers. It is easily verified that this particular  $\mathbf{Q}_m$  satisfies the constraints of (4).

The optimal rate in (4) can therefore be bounded below by

$$R^* \geq \sum_{m=1}^{N/B} \sum_{b=1}^B \mathbb{E} \left[ \log \left| \mathbf{I} + \rho \hat{\mathbf{H}}_{m,b} \left[ \sum_{b'=1}^B \alpha_{m,b'} \mathbf{Q}_{m,b'} \right] \hat{\mathbf{H}}_{m,b}^H \right| \right].$$

Jensen's inequality and concavity of  $\log \det(\cdot)$  imply

$$R^* \geq \sum_{m=1}^{N/B} \sum_{b=1}^B \sum_{b'=1}^B \alpha_{m,b'} \mathbb{E} \left[ \log \left| \mathbf{I} + \rho \hat{\mathbf{H}}_{m,b} \mathbf{Q}_{m,b'} \hat{\mathbf{H}}_{m,b}^H \right| \right].$$

Assuming that the channels are i.i.d., we conclude

$$R^* \geq \frac{N}{B} \mathbb{E} \left[ \log \left| \mathbf{I} + \rho \hat{\mathbf{H}}_{m,b} \mathbf{Q}_{m,b} \hat{\mathbf{H}}_{m,b}^H \right| \right] + \sum_{m=1}^{N/B} \sum_{b=1}^B \sum_{\substack{b'=1 \\ b' \neq b}}^B \alpha_{m,b'} \mathbb{E} \left[ \log \left| \mathbf{I} + \rho \hat{\mathbf{H}}_{m,b} \mathbf{Q}_{m,b'} \hat{\mathbf{H}}_{m,b}^H \right| \right].$$

Furthermore, using majorization theory [20], it can be shown that the bound above is further lower bounded for  $\alpha_{m,b'} = \frac{1}{B}, \forall m, b'$ . Again using the i.i.d. property of the channels, all terms are now independent of the summation indices, and we have

$$R^* \geq \frac{N}{B} \mathbb{E} \left[ \log \left| \mathbf{I} + \rho \hat{\mathbf{H}}_{m,b} \mathbf{Q}_{m,b} \hat{\mathbf{H}}_{m,b}^H \right| \right] + \frac{N(B-1)}{B} \mathbb{E} \left[ \log \left| \mathbf{I} + \rho \mathbf{H}_{m,b} \mathbf{Q}_{m,b'} \mathbf{H}_{m,b}^H \right| \right]. \quad (6)$$

The last term originally contained channel estimates  $\hat{\mathbf{H}}_{m,b}$  and a mismatched covariance matrix  $\mathbf{Q}_{m,b'}$ , since  $b' \neq b$ . We have replaced  $\hat{\mathbf{H}}_{m,b}$  by  $\mathbf{H}_{m,b}$ , which is possible because both of them have the same distribution and are independent of  $\mathbf{Q}_{m,b'}$ . Thus the last term is not affected by channel inaccuracies. Further, the loss due to mismatched  $\mathbf{Q}_{m,b'}$  and  $\mathbf{H}_{m,b}$  is particularly high for low SNR values, but negligible for high SNR.

Viewed as a function of  $B$ , the lower bound is

$$R^* \geq R_{\text{LB}}(B) = \frac{N}{B} c_1 + \frac{N(B-1)}{B} c_2, \quad (7)$$

where  $c_1$  and  $c_2$  are the expected values in (6). While  $c_2$  is independent of the channel inaccuracy  $\alpha$ ,  $c_1$  is a function of  $\alpha$  in a way described below.

We will use  $R_{\text{LB}}(B)$  as a guideline for choosing  $B$ . For now, note that  $R_{\text{LB}}(B)$  is monotonously decreasing in  $B$  for  $c_1 > c_2$ , and monotonously increasing if  $c_1 < c_2$ .

First, consider the case of accurate channel knowledge. Since the covariance matrix  $\mathbf{Q}_{m,b}$  in the first term of (6) by definition matches the channel  $\mathbf{H}_{m,b}$ , the log term is maximized by this particular transmit covariance matrix. Specifically, the expected value in the first term exceeds the expected value in the second term, i.e.,  $c_1 > c_2$ . Thus,  $R_{\text{LB}}(B)$  is monotonously decreasing, and therefore  $B = 1$  is the optimal chunk size.

Let us now consider the case of outdated or inaccurate channel information. To evaluate  $c_1$  (the first term in (6)), we will first need an explicit expression for  $\mathbf{Q}_{m,b}$ . To this end, denote the singular value decomposition of  $\mathbf{H}_{m,b}$  as  $\mathbf{H}_{m,b} = \mathbf{U}_{m,b} \mathbf{\Sigma}_{m,b} \mathbf{V}_{m,b}^H$ , where  $\mathbf{U}_{m,b}$  and  $\mathbf{V}_{m,b}$  are unitary matrices and  $\mathbf{\Sigma}_{m,b}$  is a diagonal matrix with sorted elements  $\sigma_{m,b}^{(1)} \geq \dots \geq \sigma_{m,b}^{(n_T)}$ . Then we can write  $\mathbf{Q}_{m,b} = \mathbf{V}_{m,b} \mathbf{P}_{m,b} \mathbf{V}_{m,b}^H$ , where  $\mathbf{P}_{m,b}$  is a diagonal matrix with non-negative entries  $p_{m,b}^{(1)}, \dots, p_{m,b}^{(n_T)}$ . These represent the optimal power loading, obtained from the water-filling algorithm [19] with total power budget  $P/N$  and streamwise signal-to-noise ratios  $\{\rho(\sigma_{m,b}^{(i)})^2 \mid i = 1, \dots, n_T\}$ .

According to the first term in (6),  $\mathbf{Q}_{m,b}$  is now used as transmit covariance for the channel  $\hat{\mathbf{H}}_{m,b}$ . Using equation (3), and applying the (non-matched) linear receiver processing  $\mathbf{U}_{m,b}^H$ , it can be seen that the  $i$ th received stream will consist of a desired signal component of power  $(1-\alpha)p_{m,b}^{(i)}(\sigma_{m,b}^{(i)})^2$ , a thermal noise component of power  $1/\rho$ , and a (non-Gaussian) inter-stream interference component of power  $\alpha P/N$ . Under the reasonable assumption that the receiver treats the interference as Gaussian (worst-case) noise, the resulting SNR is  $\frac{(1-\alpha)p_{m,b}^{(i)}(\sigma_{m,b}^{(i)})^2}{1/\rho + \alpha P/N}$ . Comparing this expression to the case with

accurate channel information ( $\alpha = 0$ ), we incur an SNR loss<sup>2</sup> of

$$\Gamma = \frac{1 - \alpha}{1 + \alpha \rho P/N}.$$

(Observe that  $\Gamma(\alpha)$  is monotonously decreasing.) The power loss  $\Gamma$  characterizes the effect of inaccurate channel information on (6) completely. Namely, the first term is given through

$$c_1 = \mathbb{E} \left[ \sum_{i=1}^{n_T} \log \left( 1 + \Gamma \rho p_{m,b,i} \sigma_{m,b,i}^2 \right) \right]. \quad (8)$$

Recall that for  $\alpha = 0$ , we have  $c_1 > c_2$ , and small chunks ( $B = 1$ ) are optimal. As  $\alpha$  increases towards 1, the loss factor  $\Gamma$  decreases, and so does  $c_1$  via (8). From some critical point onwards, i.e., for  $\alpha > \alpha_{\text{crit}}$ , the relation between  $c_1$  and  $c_2$  will have switched to  $c_1 < c_2$ . For these cases, the lower bound (7) is no longer maximized by  $B = 1$  (one chunk per subcarrier). Instead, (7) predicts that  $B$  should be as large as possible, namely  $B = N$  (one chunk for all subcarriers). In practice however, the lower bound is not always a good approximation of the actual  $R^*$ . We thus expect the optimal  $B$  to increase gradually from  $\alpha_{\text{crit}}$  onwards, instead of jumping from 1 to  $N$  immediately.

#### IV. MULTI-USER CASE

Similarly to the single-user case, we need to solve the following optimization problem.

$$\text{maximize} \quad \sum_{m=1}^{N/B} \sum_{b=1}^B \mathbb{E} \left[ \log \left| \mathbf{I} + \sum_{k=1}^K \rho \hat{\mathbf{H}}_{m,b,k} \mathbf{Q}_{m,k} \hat{\mathbf{H}}_{m,b,k}^H \right| \right], \quad (9)$$

subject to  $\mathbf{Q}_{m,k} \succeq \mathbf{0}$ ,  $\forall k$

$$\sum_{m=1}^{N/B} \text{tr}(\mathbf{Q}_{m,k}) \leq \frac{P}{B}, \quad \forall k,$$

where the optimization is over the  $\mathbf{Q}_{m,k}$ , i.e., the transmit covariance in the  $m$ th chunk for the  $k$ th user, for  $m = 1, \dots, N/B$  and  $k = 1, \dots, K$ . The problem data are  $N$ ,  $B$ ,  $P$ ,  $\rho$ , and  $\hat{\mathbf{H}}_{m,b,k}$ , the inaccurate channel states for the  $b$ th subcarrier of the  $m$ th tile for the  $k$ th user. Optimization (9) can be solved by an efficient algorithm based on an outer iterative water filling [10] and an inner fix-point algorithm as in the single-user case. The following update rule [17] should be used for every user and every chunk (we drop the indices  $m$  and  $k$  here for brevity):

$$\mathbf{Q}^{\ell+1} = \frac{\mathbb{E} \left[ P \sum_{b=1}^B \left( \mathbf{Q}^{\ell,1/2} \mathbf{H}_b^H \mathbf{G}_b \mathbf{H}_b \mathbf{Q}^{\ell,1/2} \right) \right]}{\mathbb{E} \left[ \text{tr} \sum_{b=1}^B \left( \mathbf{Q}^{\ell,1/2} \mathbf{H}_b^H \mathbf{G}_b \mathbf{H}_b \mathbf{Q}^{\ell,1/2} \right) \right]}$$

where

$$\mathbf{G}_b = \left[ \mathbf{Z}_b + \rho \mathbf{H}_b \mathbf{Q}^{\ell} \mathbf{H}_b^H \right]^{-1}.$$

<sup>2</sup>The power loss  $\Gamma$  can be interpreted as two separate effects. Firstly, the variance of the (effective) channel gains reduces from 1 to  $1 - \alpha$ . Secondly, the noise power increases from  $\sigma_n^2$  to  $\sigma_n^2 + \alpha \frac{P}{N}$ , due to residual inter-stream interference.

The initialization can be chosen as  $\mathbf{Q}^0 = \frac{P}{n_T} \mathbf{I}$ . Here,  $\mathbf{Z}_b$  is the spatial noise plus interference covariance matrix. Following the lines of arguments as in the single-user case, we arrive at

$$R^* \geq \frac{N}{B} \mathbb{E} \left[ \log \left| \mathbf{I} + \sum_{k=1}^K \rho \hat{\mathbf{H}}_{m,b,k} \mathbf{Q}_{m,b,k} \hat{\mathbf{H}}_{m,b,k}^H \right| \right] + \frac{N(B-1)}{B} \mathbb{E} \left[ \log \left| \mathbf{I} + \sum_{k=1}^K \rho \hat{\mathbf{H}}_{m,b',k} \mathbf{Q}_{m,b',k} \hat{\mathbf{H}}_{m,b',k}^H \right| \right]. \quad (10)$$

Here, the  $\mathbf{Q}_{m,b,k}$  are the per-subcarrier optimal transmit covariances, and thus matched to the channels  $\mathbf{H}_{m,b,k}$ . They are obtained through iterative water filling [10] on each subcarrier.

Due to the similar form of this expression to the one in (6), the effects of imperfect channel knowledge are comparable to the singular case. A detailed discussion is thus omitted.

#### V. SIMULATION RESULTS

In Fig. 2, the data rates for different tile sizes is depicted as a function of the parameter  $\alpha$  for SNR = 5 dB for the single user case. All rates are normalized with respect to the best scenario, i.e., the channel knowledge is accurate ( $\alpha = 0$ ) and the transmit covariance matrices are computed for each subcarrier separately ( $B = 1$ ). The number of antennas at the mobile as well as the base were set to  $n_T = n_R = 2$  antennas, the number of subcarriers and channel taps was  $N = 1024$  and  $L = 118$ , respectively. We observe that the performance of the system with subcarrier-wise optimization,  $B = 1$ , degrades severely for increasing  $\alpha$ . Furthermore, the higher the tile size, the more robust is the system with respect to channel changes, which confirms the insight gained from the examination of the lower bound. Thus, higher tile sizes not only reduce the computational complexity and feedback overhead (one covariance matrix for all subcarriers within one tile), but also provide *robustness*.

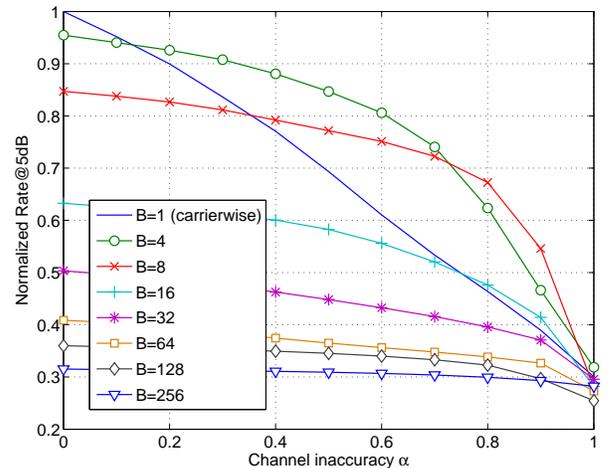


Fig. 2. Average normalized rates for different tile sizes.  $L = 118$ ,  $n_T = n_R = 2$ ,  $N = 1024$ , SNR = 5 dB

We observe that a tile size of  $B = 4$  outperforms the carrierwise optimization case at  $\alpha \approx 0.15$ , and a tile size

of  $B = 8$  outperforms the carrierwise case at  $\alpha \approx 0.35$ , and beats all other curves at around  $\alpha = 0.8$ .

In Fig. 3, the normalized data rates are now depicted for a SNR = 20 dB for the same setup. We observe that the decline in performance in case of  $B = 1$  is not as pronounced as for low SNR. However, the normalized data rates achievable for higher tile sizes have increased, which again confirms the insight gained from the examination of the lower bound. From the figure, we observe that at 20 dB, higher tile sizes outperform the carrier-wise optimization at slightly lower values of  $\alpha$  than at 5 dB.

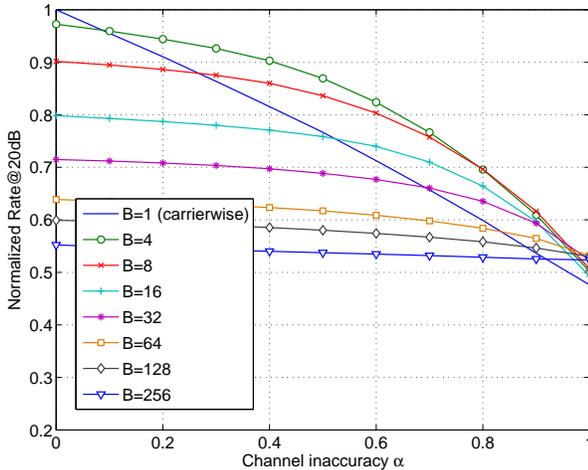


Fig. 3. Average normalized rates for different tile sizes.  $L = 118$ ,  $n_T = n_R = 2$ ,  $N = 1024$ , SNR = 20 dB

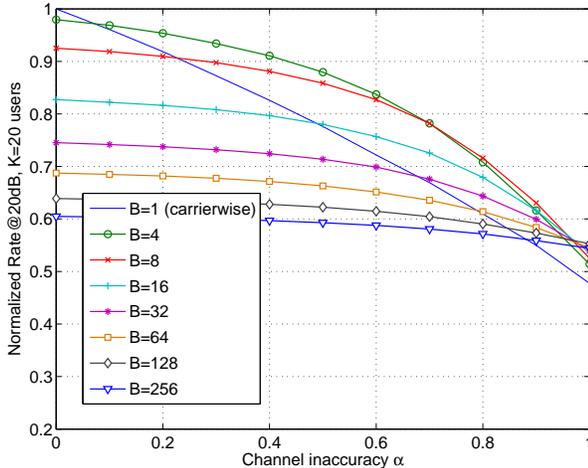


Fig. 4. Multiuser: Average normalized rates for different tile sizes.  $K = 20$ ,  $L = 118$ ,  $n_T = n_R = 2$ ,  $N = 1024$ , SNR = 20 dB

In Fig. 4, the normalized data rates are shown for the multi-user case at SNR = 20 dB. While the performance for  $B = 1$  (carrierwise) has not changed at all versus the single-user case, the normalized data rates achievable for higher tile sizes have slightly increased, more dominantly for higher tile sizes. From the figure, we observe that at  $\alpha = 0.5$ , even the system with tile size  $B = 16$  is outperforming the carrier-wise optimization.

## VI. CONCLUSION

Complexity and overhead are two of the main practical drawbacks in advanced MIMO transmission techniques.

Bundling resources into tiles has been proposed as a means to reduce both of them, but often comes at the cost of reduced performance. However, under the realistic assumption of uncertainty in the channel state information, we have shown that tile-based processing is able to introduce *robustness* to the system. Larger tile sizes can in fact improve performance. In these cases, tile-based processing is a win-win situation, since it can reduce complexity, decrease required feedback, and improve performance, all at the same time.

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