Wave Digital Information Anticipator

Karlheinz Ochs, Eloy Hernandez-Guevara, and Enver Solan

Ruhr-University Bochum
Chair of Digital Communication Systems
Bochum, Germany
Email: \{Karlheinz.Ochs, Enver.Solan\}@rub.de

Abstract—Pioneering developments in electrical engineering are based on inspirations from biology. They exhibit naturally an efficient information processing. For example, hardware realizations of memristive circuits mimicking the anticipatory behavior of unicellular organisms like amoebas have been developed in this context. Unfortunately, circuits are not appropriate for algorithms dedicated in the area of digital signal processing. We intended to get an algorithmic model of the anticipation circuit for utilizations in digital signal processing applications. In our approach we have applied the wave digital method to get a digital replica of the analog circuit. This offers several benefits, like the ability for preserving the passivity of the analog counterpart, the possibility for a parallel processing approach, efficiency, and robustness of the resulting algorithmic model. The memristive device of the original circuit has been replaced by a novel multilevel memristor model with suitable features for anticipation of digital patterns. The resulting algorithmic model offers innovative applications in e.g. robotics or artificial neural networks.

I. INTRODUCTION

Even in simple unicellular organisms, like amoebas, fundamental information processing approaches have been observed [1]. As it turned out, these organisms have the ability for the anticipation of periodic events. In order to mimic efficient biological information processing systems, e.g. the capability for information anticipation, electrical circuits with similar functionalities have been developed. Several possibilities in this area have arisen since a novel elementary passive device namely the memristor [2], which is essentially a nonlinear resistor with memory, was realized in the HP-laboratories in 2008, cf. [3]. For example, in [4] a memristive oscillator, which mimics the ability for anticipation of an amoeba is presented. Recently, this circuit, capable of the anticipation of unipolar pulses, has been extended to a circuit for the anticipation of bipolar pulses [5] considering general information represented by arbitrary digital patterns. It can be used either in usual circuit simulation tools or as a hardware realization in real-time applications. However, simulation tools are not suitable for utilization in digital signal processing applications.

Our intention is to establish an algorithmic model of the anticipation circuit in order to achieve an anticipation block as a partial processing unit of a more complex algorithm. Therefore, the wave digital method [6] has been exploited. This approach was originally applied to get a digital replica of an analog filter by using a discretization method, which preserves the passivity of the analog counterpart [7]. This yields highly robust and efficient algorithms [8] and additionally a parallel implementation approach [9]. Besides, this method also allows for reproducible analyses [10] in contrast to real measurements, which are exposed to parameter spread. For instance, in [11], a method for emulation of real electrical components based on wave digital methods is shown. We present here a wave digital realization of the proposed anticipatory system. The resulting algorithm is used in simulations for anticipation of images as information carriers. Since this method is particularly suitable for emulations on embedded systems, the resulting algorithm can also be utilized for implementations on digital signal processors as well as on application-specific integrated circuits.

In the further course of this work, a novel memristor model is introduced. Then, the transformation of the reference circuit into the wave digital domain, after a short recapitulation of the functionality, is described. Afterwards, the extension from a single anticipator to a field of anticipators is explained. Respective sections are attended by corresponding simulation results. A conclusion summarizes the main results.

II. MULTILEVEL MEMRISTOR MODEL

The memristive device in the anticipatory circuit is of great importance regarding the capability for learning because it represents the memory of the system. In contrast to the original circuit in [5], a novel voltage-controlled multilevel memristor model with appropriate features is introduced. For the sake of clarity, we consider here the memductance $W$ instead of the memristance $M$, with $W = 1/M$. It is modeled especially for the anticipatory circuit and can be described by

$$i_m(t) = W(z) u_m(t),$$

with

$$\frac{dz}{dt} = f(u_m) = u_m(t) + U_o.$$ (2)

Here, $U_o = 50$ mV characterizes a retention and $z$ denotes the internal state. Since it is mathematically the integral of a voltage, it has the dimension of a magnetic-flux. The memris-
This memductance vs. state map offers several benefits in particular for the functionality of the anticipatory system. First of all, we can control the number of resistive switching levels by $n$, which influences the speed of memductance change. For simulations in this work $n = 5$ has been chosen, as depicted in Fig. 1. The higher this number is, the more continuously is the change between different states. Secondly, the distance between two separate states is given by $\Delta z = 0.7$ Wb. This is also a parameter for the learning procedure and can be considered as a threshold for changing the conductance. Lastly, the asymmetric shape achieves decoupling between positive pulses for anticipating 1 and negative pulses for anticipating 0, respectively. The concrete relation between the memristor model and the anticipatory system is explained in the next section. It should be emphasized that the presented memristive system can be replaced by other devices based on different physical and chemical phenomena [12].

III. WAVE DIGITAL BIT ANTICIPATOR

In this section, a modified version of the anticipation circuit is briefly recapitulated, cf. Fig. 2 a). The interested reader is referred to [5] for more details and a deeper analysis of the circuit. It mainly consists of a $RLC$ circuit representing biological oscillators and dissipative processes within the amoeba in combination with a memristive device acting as a memory. In contrast to the original circuit, an ideal transformer is introduced in order to anticipate positive ($n = -1$) as well as negative ($n = 1$) pulses representing bit values of 1 and 0, respectively. Therefore, the amplitude of the oscillating voltage $u_m(t)$ over the memristive device is evaluated. The main mechanism depends on the fact that oscillating voltages over the memristive device increase or decrease its conductance depending on the sign and amplitude. Low memductance values lead to an oscillating output voltage with a larger amplitude compared to high memductance values.

In order to get a wave digital model of this circuit, it has to be decomposed into its components, i.e. the circuit elements as well as interconnections. In doing so, we can make use of the locality principle of the wave digital method [6] to transfer each component individually and compose them subsequently. The independent variable, namely the time $t$, is discretized into time instants $t_k = t_0 + kT$, $k \in \mathbb{N}_0$, where $T$ denotes the sampling period and $t_0$ is the initial time. This transformation procedure inherently transforms the underlying differential equations into difference equations. Once the system is described in the time-discrete domain, the resulting signal parameters are expressed by means of wave quantities instead of voltages and currents. Voltages and currents are related to wave quantities through the bijective mapping

$$a = u + R' i \quad \text{and} \quad b = u - R' i, \quad \text{with} \quad R' \geq 0.$$  \hspace{1cm} (4)

Here, $a$ denotes the incident wave at one particular port and $b$ the reflected wave. The corresponding port resistance $R'$ has an arbitrary but positive value. Using wave quantities as signal parameters is preferable because it conveys implicit expressions into explicit ones [6] as illustrated for a capacitance. The difference equation of a capacitance considering the trapezoidal rule for the numerical integration is

$$\frac{u_C(t_k)}{T} \approx \frac{u_C(t_{k-1})}{2C} \left[ i_C(t_k) + i_C(t_{k-1}) \right]. \hspace{1cm} (5)$$

Note, this is an implicit equation since $u_C(t_k)$ depends on $i_C(t_k)$ and the latter via the electrical network on $u_C(t_k)$. Exploiting equation (4) and interpreting $R' = R_C = T/[2C]$ as port resistance leads to

$$b_C(t_k) \approx a_C(t_{k-1}), \hspace{1cm} (6)$$

which is explicit and therefore more suitable for implementations. Equation (6) can be realized by a delay element as shown in Fig. 2 c) and d). The inductance is the dual element of the capacitance and consequently a similar derivation yields

$$b_L(t_k) \approx -a_L(t_{k-1}), \quad \text{with} \quad R' = R_L = \frac{2L}{T}. \hspace{1cm} (7)$$

cf Fig 2 c) and d). From

$$e(t) = u(t) - R i(t) \hspace{1cm} (8)$$

for the resistive voltage source, we get

$$e(t_k) = a(t_k) \quad \text{with} \quad R = R' \hspace{1cm} (9)$$

in the wave digital domain, which is also shown in Fig. 2 a) and c). The remaining correspondence for the ideal transformer is described by

$$u_m(t) = n u_p(t) \quad \text{and} \quad i_p(t) = -n i_m(t) \hspace{1cm} \Rightarrow \hspace{1cm} \begin{cases} b_p(t_k) &= n a_m(t_k) \\ b_m(t_k) &= n a_p(t_k) \end{cases}, \hspace{1cm} (10)$$

for $n \in \{-1, 1\}$, cf. Fig 2 a) and c). The direction of voltages and currents are chosen with respect to port conditions.
on adaptor ports regarding Kirchhoff’s voltage and current equation. For the interconnection of individual elements, corresponding wave digital components representing Kirchhoff’s series and parallel interconnections are needed. In the wave digital domain, such interconnections are realized by series and parallel adaptors, respectively. In Fig. 2 a) and c), both adaptors are depicted with corresponding port resistances and adaptor coefficients

\[
\gamma_s = \frac{R}{R_s}, \quad \text{with} \quad R_s = R + R_L \quad (11)
\]

\[
\gamma_p = \frac{R_p}{R_s}, \quad \text{with} \quad R_p = \frac{R_0 R_C}{R_s + R_C} \quad (12)
\]

The signal flow diagrams of both adaptors can be found elsewhere, e.g. [6].

Wave digital realizations of memristive devices need a more precise investigation [11], [13]. For a wave digital realization, a corresponding electrical reference circuit is needed. One possible reference circuit of a memristive system is shown on the right-hand side of Fig. 2, where the input-output relation is represented by the memristive one-port a) and the memristive behavior by the integrator circuit b). Since the memristive device exhibits variant resistances depending on the excitation in the past, it is represented by a variable reflection coefficient

\[
b_m(t_k) = \rho(W) a_m(t_k), \quad \text{with} \quad \rho(W) = \frac{1 - R_p W}{1 + R_p W} \quad (13)
\]

in the wave digital domain, cf Fig 2 c). Due to the fact, that the reflected wave \( b_m \) as well as the reflection coefficient \( \rho \) itself depend on the excitation, equation (13) leads to an implicit equation, which can be solved iteratively for each time instant. However, sufficient small sampling periods as well as an appropriate memristor model decrease the number of needed iterations. Conditioning of the underlying equations plays a significant role in this context. A further benefit of the novel multilevel memristor model (3) is the directly dependence to the flux. With this, we get well-conditioned equations, suitable for real-time implementations. Indeed, for simulations represented here, no iteration has been applied considering a sampling period of \( T = 10 \text{ ms} \).

For a complete wave digital realization, the integrator circuit in Fig. 2 b) also has to be transformed into the wave digital domain

\[
a_I(t_k) = T f(z) + b_I(t_k). \quad (14)
\]

A more general approach for realizing memristive systems in the wave digital domain is explained in detail in [11], [13].

**Simulation Results**

The wave digital model of the memristive oscillator leads to an algorithmic model consisting of elementary mathematical operations, like adders, multipliers, and delay elements. Transient behavior of both the input pulses as well as the oscillating output voltages is shown in Fig. 3. A good coincidence with the simulation from [5] can be observed. Moreover, the anticipation behavior is noticeable: After a training sequence of three pulses, one pulse is enough for anticipating the training sequence. This can also be interpreted as a kind of pattern recognition. For evaluation of the memductance vs. state map, the resulting state variable for each instant is needed. In Fig. 4 the transient behavior of both the state itself and of the memductance corresponding to the signals depicted in Fig. 3 is shown. Whenever the state undercuts the threshold \( \Delta z \), the memductance switches into the next low state, as excepted. For the sake of briefness, we have presented the results for positive pulses. However, negative pulses lead to similar results.
IV. ANTICIPATION OF INFORMATION

So far the transformation of the memristive reference circuit into the wave digital domain has been discussed. As it has been shown, this reference circuit is capable of anticipating single bits. In order to anticipate general digital patterns, one can utilize a field of such a single anticipator. Because of resulting efficient algorithms and adequacy of the wave digital method regarding parallel processing approaches, the algorithm is still fast and efficient independent of the exploited platform. For illustration purposes, images as information carriers have been used in simulations represented here. Therefore, the images are divided into pixels representing a single bit. A bit value of 0 is represented by a black color, whereas a 1 bit pixel is white. The anticipation of images under ideal as well as noisy conditions is depicted in Fig. 5. Consistently to the simulation results of Fig. 3, the anticipation behavior has also been observed for information represented by digital patterns as input. We have made an additional interesting observation for the noisy case: It seems that the system inherently reduces the noise influence. Depending on the number of training pulses the image becomes more clear even for higher noise levels. We know intuitively that biological systems exhibit always noisy signals and parameter spread. Nevertheless, they still work which is notable in the context of the anticipatory circuit and resulting wave digital algorithm.

V. CONCLUSION

In this paper, we have proposed a possibility for utilization of bio-inspired information anticipation in digital signal processing approaches. Therefore, a novel multilevel memristor model in combination with the wave digital method has been used, which yields efficient and robust algorithms capable of implementations for a parallel processing.

A memristive device with multilevel resistance states has been developed with suitable properties especially for the effective functionality of the anticipatory circuit. Wave digital simulation results have shown a good coincidence with the results of previous works regarding the anticipation behavior.

The wave digital algorithm of the anticipation system can be used as a distinct component in a library of wave digital components. For example, it is particularly suitable as a partial processing unit on embedded systems considering more complex applications.

This approach is suited for different applications, e.g. character recognition in today’s computer systems as well as in robotics.

ACKNOWLEDGMENT

The financial support by the German Research Foundation (Deutsche Forschungsgemeinschaft - DFG) through FOR 2093 is gratefully acknowledged.

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